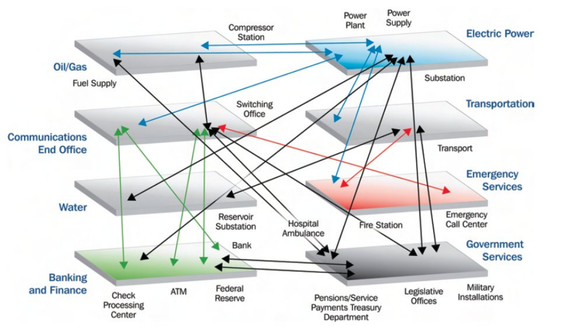

FIGURE 3.1 Connections and interdependencies across the economy. Schematic showing the interconnected infrastructures and their qualitative dependencies and interdependencies. SOURCE: Department of Homeland Security, National Infrastructure Protection Plan, available at http://www.dhs.gov/xprevprot/programs/editorial_0827.shtm.

KnowEscape2013

Espoo, 18 November 2013

Multiplex PageRank

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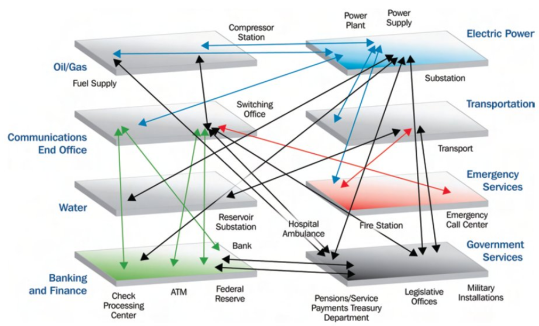


FIGURE 3.1 Connections and interdependencies across the economy. Schematic showing the interconnected infrastructures and their qualitative dependencies and interdependencies. SOURCE: Department of Homeland Security, National Infrastructure Protection Plan, available at http://www.dhs.gov/xprevprot/programs/editorial_0827.shtm.

The function of many complex

social, technological and transportation systems

depends on the non-trivial interactions between

interacting networks

Interacting Transportation networks

Transportation networks are a major example of interacting networks.

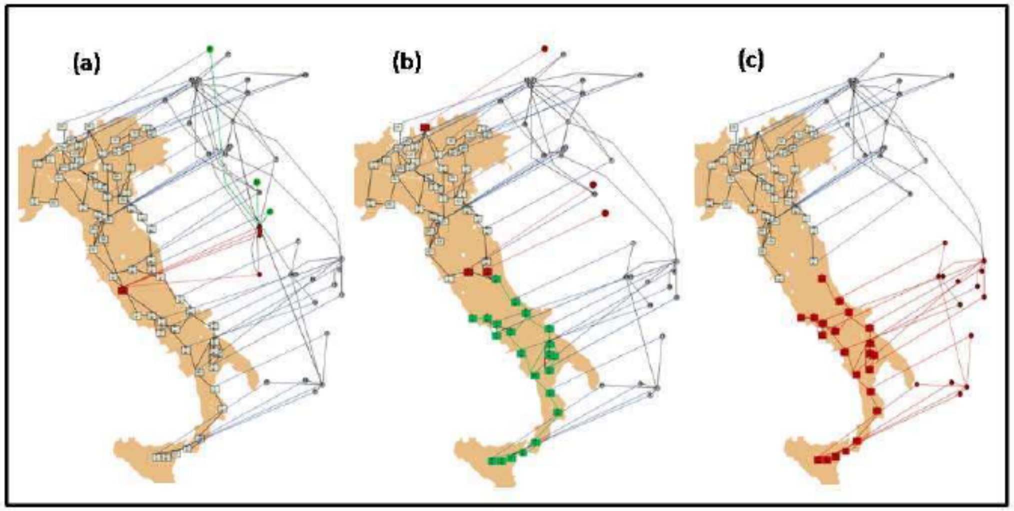
Here
blue lines represent short-range commuting flow by car or train
the red lines indicate airline flow for few selected cities

B. GONÇALVES ET AL., INDIANA UNIV.

Vespignani Nature 2010

Interacting infrastructure networks

Complex infrastructures are interdependent and with very important implications in their robustness. Which are the most essential nodes for the stability of these systems?



Buldyrev et al. Nature 2010

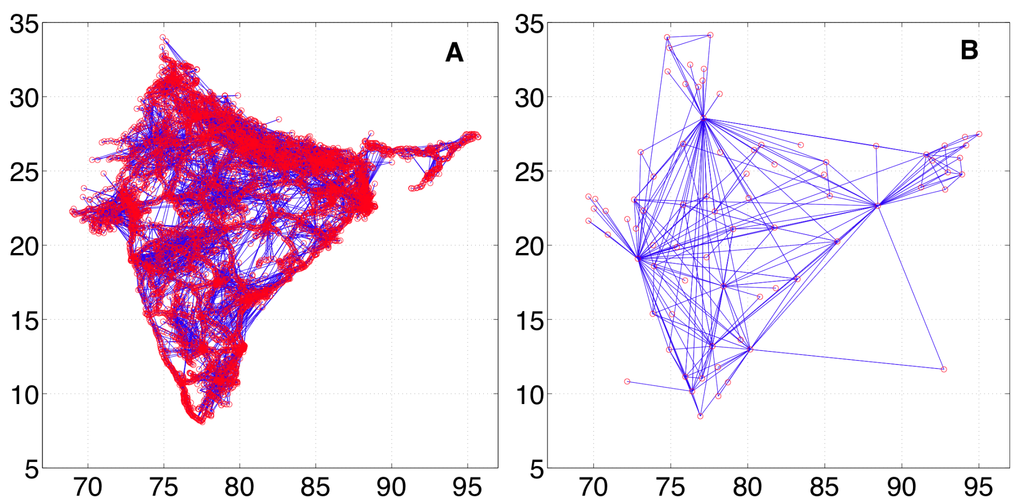
Interacting
Social networks

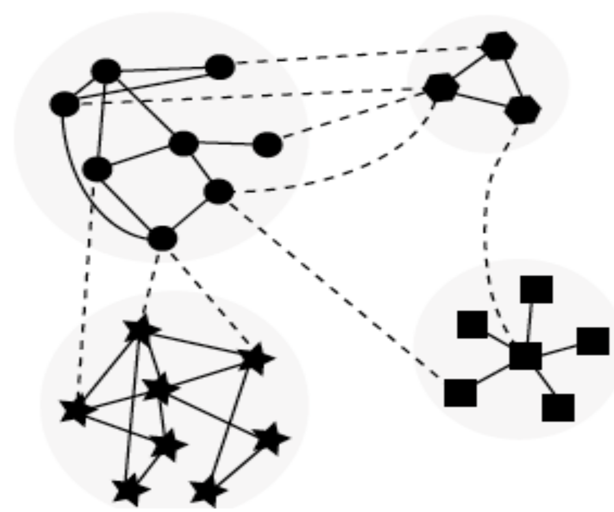
Social networks are interacting and overlapping with profound implications for community detection algorithms

Y.Y. Ahn et al. Nature 2010

Interacting networks

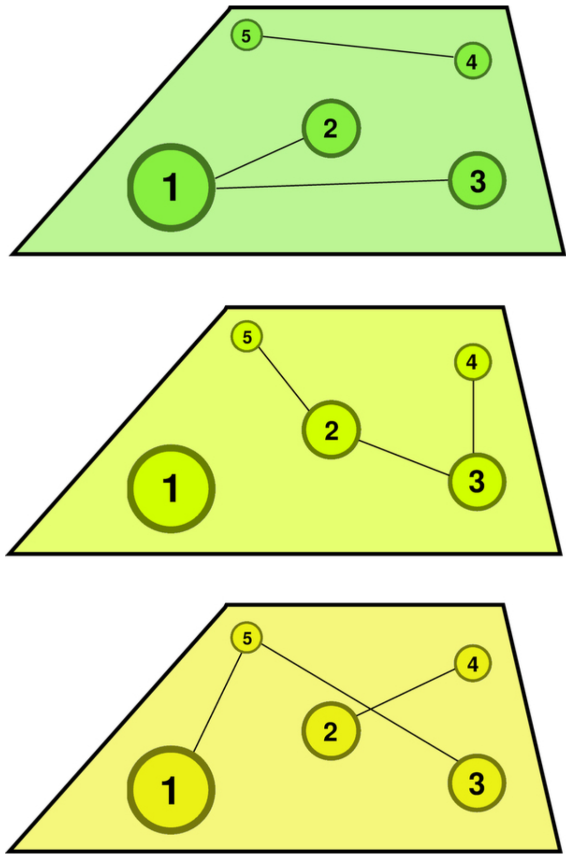
Two or more interacting networks are formed by different nodes (ex. Railways and airports networks) but there might be complex interactions and interdependencies between the nodes.





Multiplex

- A multiplex is formed by a set of nodes that are present at the same time on different networks,
- A multiplex is formed by M layers (in the figure M=3)
- Each layer is formed by a network



Representation of a duplex

The straightforward representation a multiplex of N nodes formed by 2 layers is

by means of the set of 2 adjacency matrices

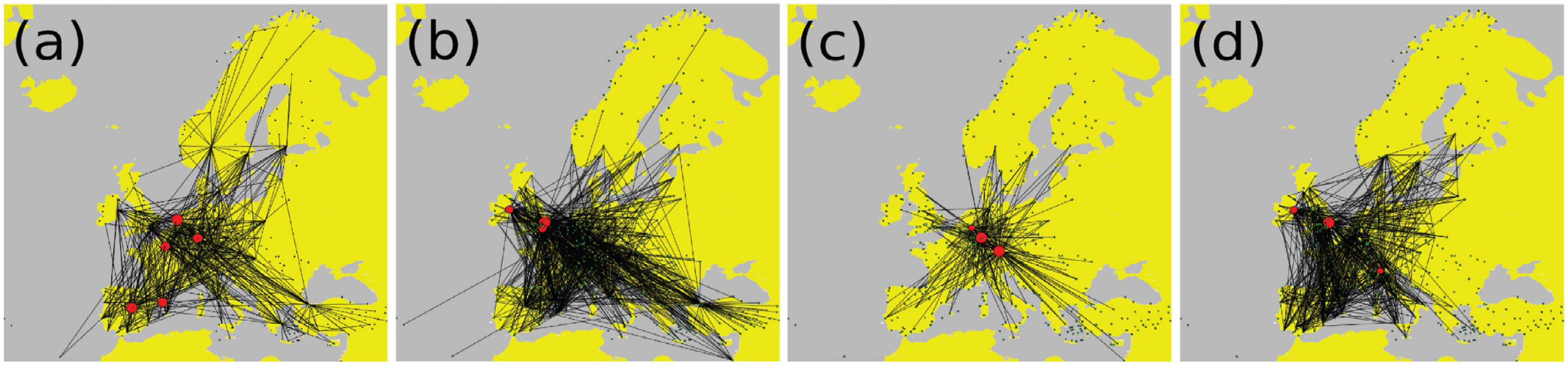
$$\boxed{A, B}$$

of matrix elements

$$A_{ij} = \begin{cases} 1 & if \;\; node \;\; j \;\; links \; to \; node \; i \;\; in \; network \;\; A \\ 0 & otherwise \end{cases}$$

$$B_{ij} = \begin{cases} 1 & if \;\; node \;\; j \;\; links \; to \; node \; i \;\; in \; network \;\; B \\ 0 & otherwise \end{cases}$$

The airport network is a multiplex

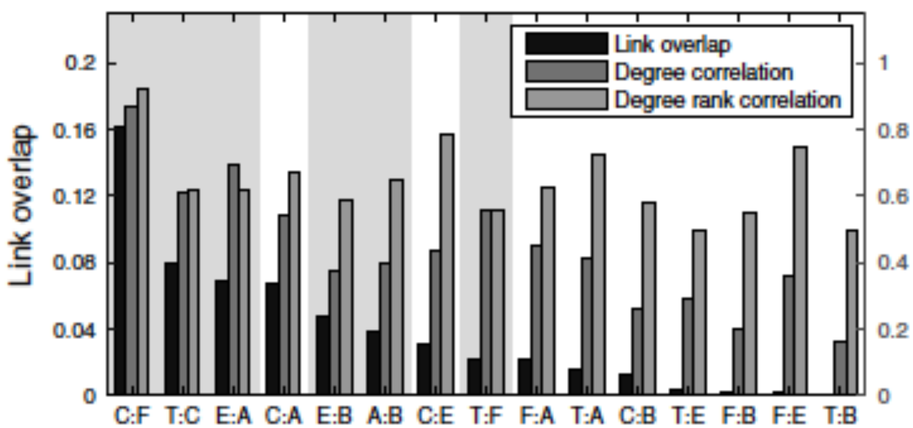


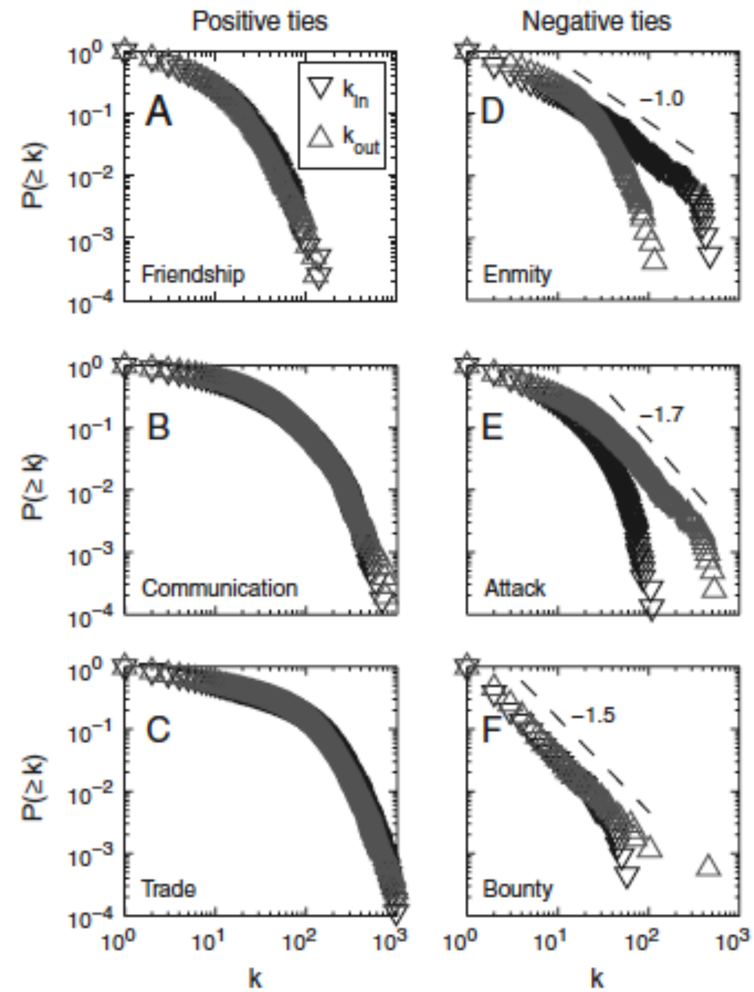
- (a) Only links belonging to all airline companies are plotted
- (b) The combined network where only nodes of degree k>75 have been plotted
- (c) A major airline network
- (d) Low cost airline network

Cardillo et al. Scientific Reports (2013).

The in silico multiplex social social network of an online game

- In this online game agents can belong to different networks Friendship, Communication, Trade, Enmity, Attack and Bounty networks





Szell et al. PNAS 2010

Extracting information from Interacting and Multiplex networks

The information encoded in

Interacting and Multiplex Networks

cannot be appreciated if we look at

single networks

therefore we need to develop new tools to

extract relevant information and rank nodes

in the new generation of network datasets formed by

networks of networks

PageRank

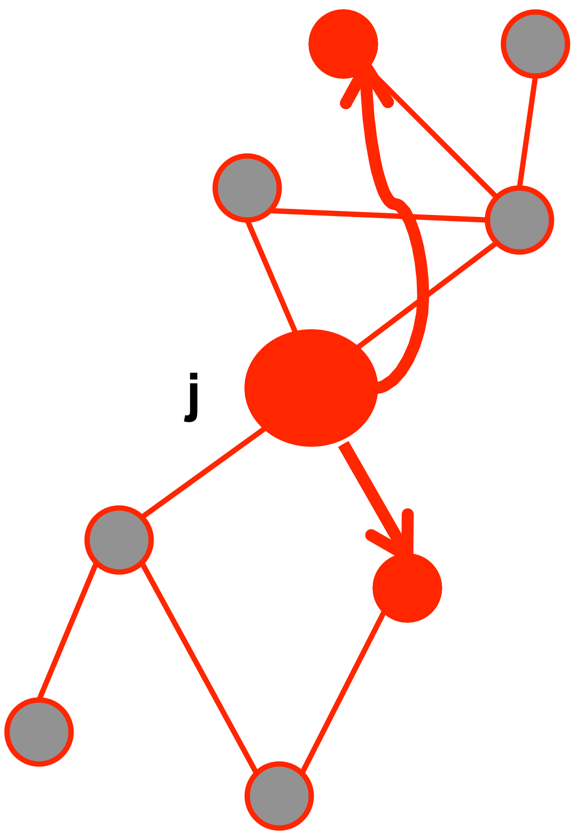
The success of

Google™

depends essentially on the exceptional performance of its ranking algorithm:

PageRank

The Page Rank is based on a random walk



- We assume to have a random walker on the node j of the network
- With probability α the random walker hops to a neighbor node
- With a probability $1-\alpha$ it jumps to a random node

PageRank

The PageRank x_i of node i is the probability that in the stationary state we find the random walker on node i

$$\boxed{x_i = \alpha \sum_j \frac{A_{ij}}{g_j} x_j + (1-\alpha)\frac{1}{N}}$$

$$with$$

$$g_j = \max(k_j, 1), \quad k_i \text{ indicating the degree of node i}$$

$$A_{ij} = \begin{cases} 1 \; if \;\; node \;\; j \; links \; to \; node \; i \\ 0 \qquad otherwise \end{cases}, \quad \alpha = 0.85$$

Approximating the PageRank with the in-degree

In a mean-field approximation where we assume that the network is uncorrelated, it is possible to approximate the PageRank with the in-degree of the node

$$\mathbf{k} = (k^{in}, k^{out})$$

$$\bar{x}(\mathbf{k}) = \frac{1}{N(\mathbf{k})} \sum_{i|\mathbf{k}_i = \mathbf{k}} x_i$$

$\Rightarrow$

$$\bar{x}(\mathbf{k}) = \alpha\, k^{in} \frac{1}{\langle k^{in} \rangle N} + (1-\alpha)\frac{1}{N}$$

S. Fortunato, M. Boguna, A. Flammini and F. Menczer
Algorithms and Model in the Web Graph (2008)

Multiplex PageRank

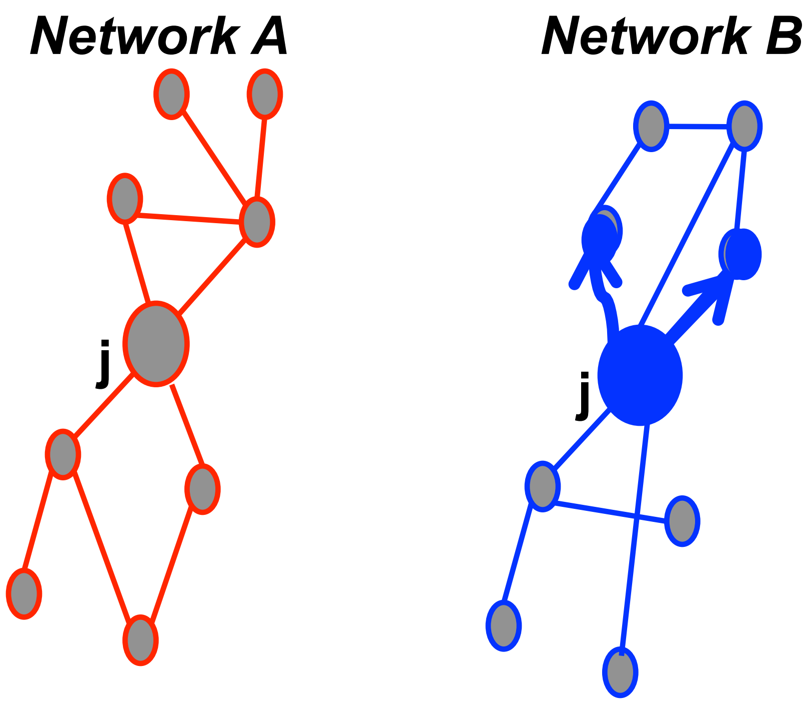
Centrality in multiplex networks

The centrality of a node in one network affects the centrality of the same node in another network

Angelina Jolie
UN Goodwill
Ambassador

For example, movie actors that become U.N. Goodwill ambassadors have a centrality in the movie actor network that affects their centrality in the socio-political network

The Multiplex PageRank is based on a biased random walk



We assume to have a random walker on the node j of the network B

With probability α the random walker hops to a neighbor node i with probability proportional to x_i^{β}

With a probability $1-\alpha$ he/she jumps to a random node chosen with probability proportional to x_i^{γ}

We assume to have a duplex, and to know the PageRank x_i of any node i in network A

Multiplex PageRank

The Multiplex PageRank X_i of node i is the probability that in the stationary state we find the random walker on network B on node i

$$\boxed{X_i = \alpha \sum_j \frac{x_i^{\beta} B_{ij}}{G_j} X_j + (1-\alpha)\frac{x_i^{\gamma}}{N\left\langle x_i^{\gamma}\right\rangle}}$$

with

$$G_j = \begin{cases} \sum_i x_i^{\beta} B_{ij} & if \quad k_j > 0 \\ 1 \quad if \; k_j = 0 \end{cases},$$

$$B_{ij} = \begin{cases} 1 \; if \;\; node \;\; j \; links \; to \; node \; i \; in \; network \; B \\ \quad 0 \qquad otherwise \end{cases}$$

Additive, Multiplicative Combined and Neutral Multiplex Pagerank

$$X_i = \alpha \sum_j \frac{x_i^{\beta} B_{ij}}{G_j} X_j + (1-\alpha) \frac{x_i^{\gamma}}{N\langle x_i^{\gamma} \rangle}$$

- Additive (β=0,γ=1)

Being central in network A enables a node to gain more centrality in network B, regardless of the node's capacity to attract important others in network network B. (Personalized vector)

- Multiplicative (β=1,γ=0)

The more important a node is in network A, the more value the node can extract from the connections received from important others in network B.

- Combined (β=1,γ=1)

A node's high centrality in network A can boost its centrality in network B both in itself and at the same time by amplifying the node's ability to derive centrality from other linked important nodes

- Neutral (β=0,γ=0)

This refers to the case in which there is no effect of network A upon network B, and thus Multiplex PageRank reduces to the PageRank based simply on network B in isolation.

Approximating the Multiplex PageRank with the degrees

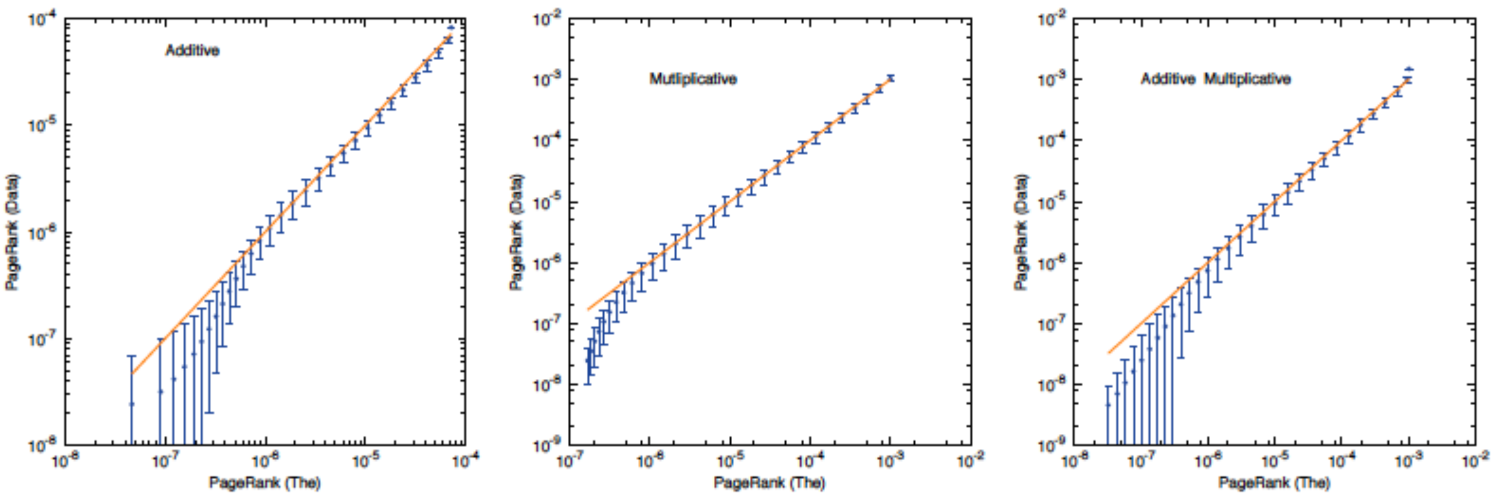
Definitions

$$\mathbf{k}_B = (k_B^{in}, k_B^{out})$$

$$\overline{X}(\mathbf{k}_B, x) = \frac{1}{N(\mathbf{k}_B, x)} \sum_{i \mid \mathbf{k}_{B,i} = \mathbf{k}_B, x_i = x} X_i$$

A mean-field calculation performed for uncorrelated B network gives

$$\overline{X}(\mathbf{k}_B, x) = \alpha x^{\beta} k_B^{in} \frac{1}{\langle x^{\beta} k_B^{in} \rangle N} + (1-\alpha) \frac{x^{\gamma}}{N \langle x^{\gamma} \rangle}$$



Approximating the Multiplex PageRanks with the degrees of the nodes

Combining the two "mean-field" approximations for x_i and X_i we get the following approximations for the Multiplex PageRank

Additive Multiplex PageRank

$$\overline{X}(\mathbf{k}_A,\mathbf{k}_B) = B_a k_B^{in} + C_a k_A^{in} + D_a$$

Multiplicative Multiplex PageRank

$$\overline{X}(\mathbf{k}_A,\mathbf{k}_B) = A_m k_A^{in} k_B^{in} + B_m k_B^{in} + D_m$$

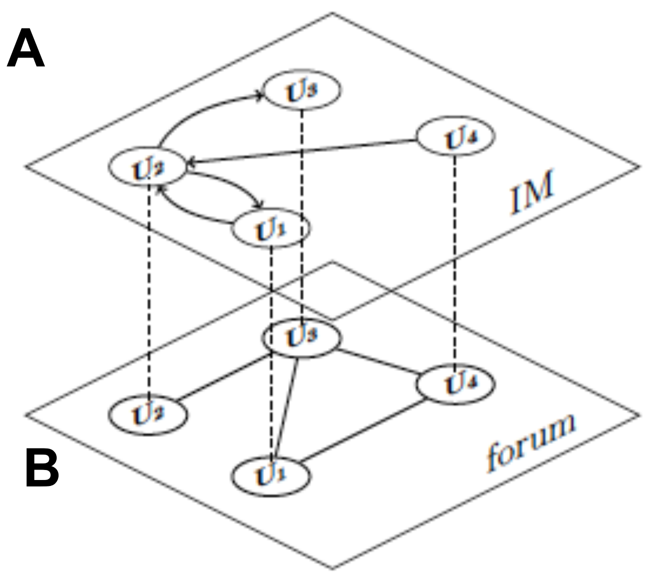
Combined Multiplex PageRank

$$\overline{X}(\mathbf{k}_A,\mathbf{k}_B) = A_c k_A^{in} k_B^{in} + B_c k_B^{in} + C_a k_A^{in} + D_a$$

Neutral Multiplex PageRank

$$\overline{X}(\mathbf{k}_A,\mathbf{k}_B) = B_n k_B^{in} + D_n$$

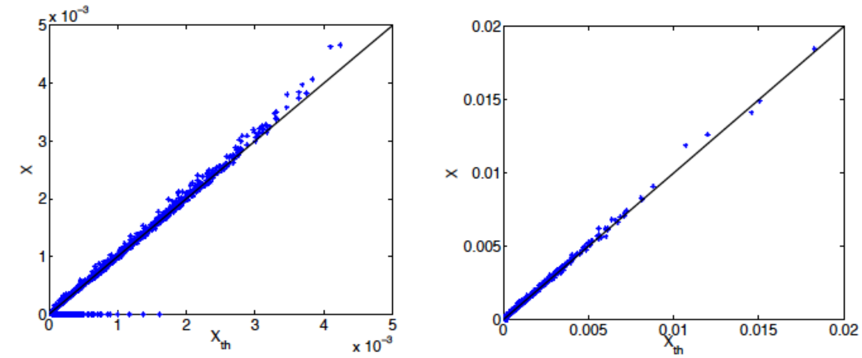
Instant Messaging and Forum Dataset at University of California Irvine



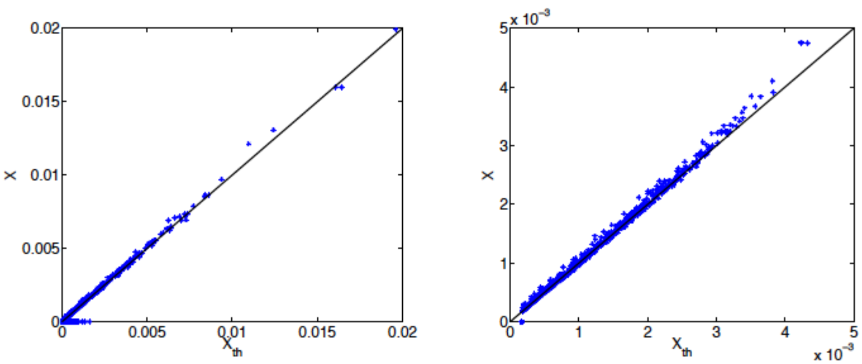
- Users in the IM 1899 active from April19 to October 26, 2004
- Users in the Forum 889,552 Thematic groups; Forum active from May 14 to October 26 2004
- Two users in the forum network are linked if they have posted on the same thematic group in a given timeframe.
- The analysis starts from June 4 2004 where both networks where operational and stable
- For every time t we have extracted the adjacency matrices A(t) and B(t) cumulating the number of interactions occurred in the last three weeks

Multiplex PageRank vs Theoretical expectation

There is a very good agreement of the MultiPlex PageRank with the theoretical expectation derived in the hypothesis of an uncorrelated Forum network



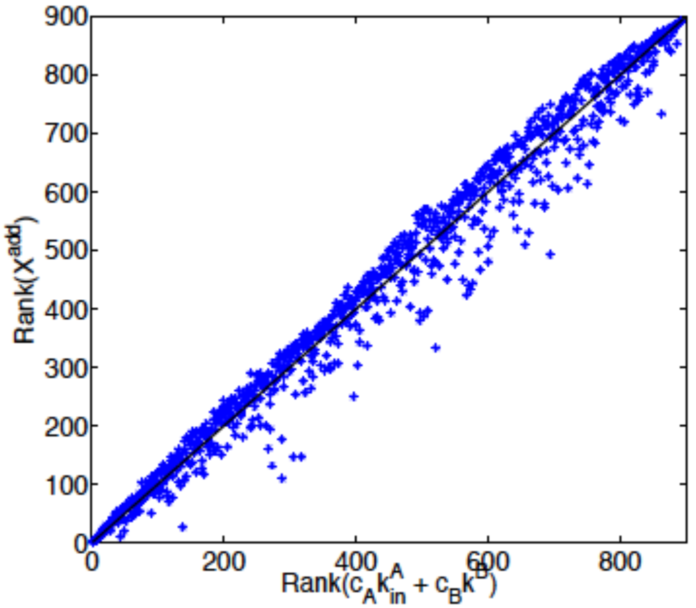
a left: additive ($\beta = 0, \gamma = 1$), right: multiplicative ($\beta = 1, \gamma = 0$)



b left: combined ($\beta = 1, \gamma = 1$), right: neutral ($\beta = 0, \gamma = 0$)

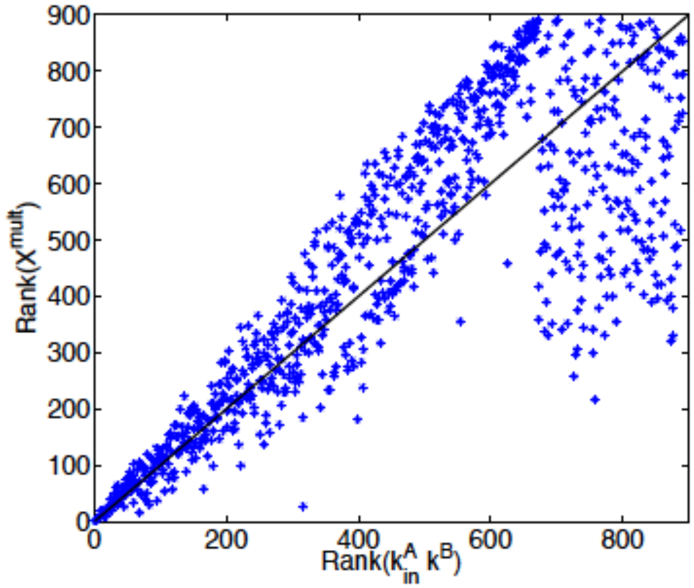
Correlation of the Multiplex PageRank with a function of the degrees

Additive PageRank



The Additive PageRank correlates with a linear combination of the in-degrees in different layers

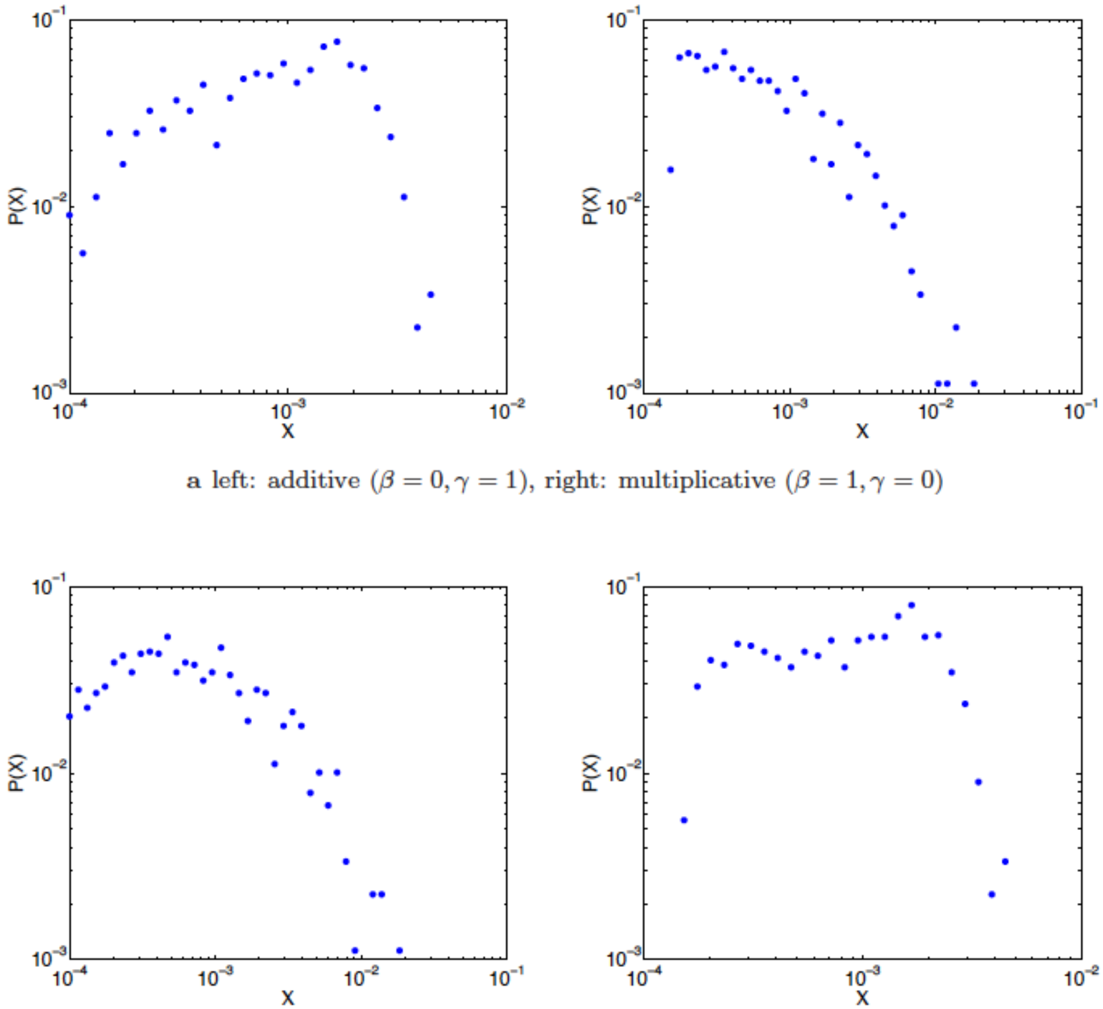
Multiplicative PageRank



The Multiplicative PageRank correlates with the product of the in-degrees in different layers

Distribution of the Multiplex PageRank

The distribution of the Multiplicative and the Combined Multiplex PageRank is broad

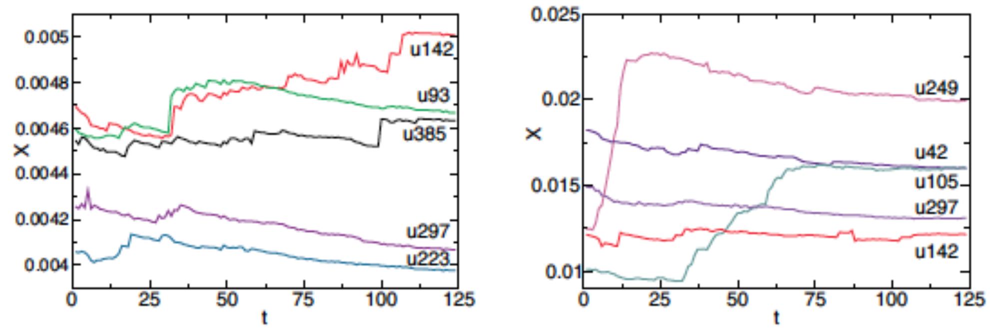


a left: additive ($\beta = 0, \gamma = 1$), right: multiplicative ($\beta = 1, \gamma = 0$)

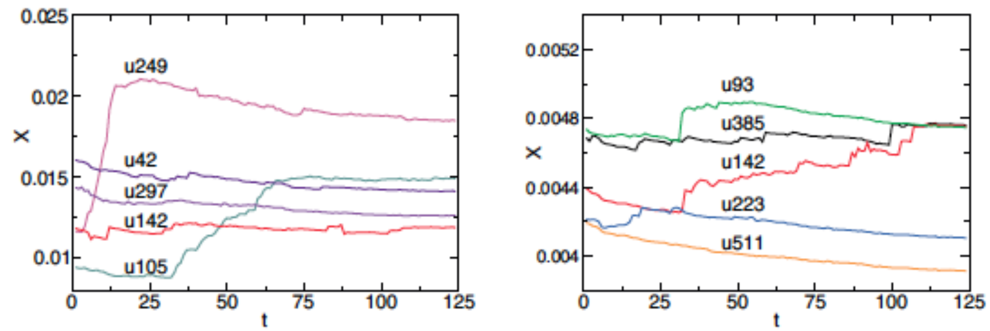
b left: combined ($\beta = 1, \gamma = 1$), right: neutral ($\beta = 0, \gamma = 0$)

Top Rank Users and Their Stability

The top rank users of the Multiplicative and Combined Multiplex PageRank are the same.
The top rank users of the Additive and Neutral Multiplex PageRank are the same with the exception of users 297 and user 511
(note that user 297 has high PageRank in the IM network)



a left: additive ($\beta = 0, \gamma = 1$), right: multiplicative ($\beta = 1, \gamma = 0$)



b left: combined ($\beta = 1, \gamma = 1$), right: neutral ($\beta = 0, \gamma = 0$)

Conclusions

- Many **networks interact, coexist and coevolve with other networks forming multiplexes** where any pair of nodes can be linked by different types of interaction.

- **The Multiplex PageRank** is the measure that evaluates how central is a node in a multiplex.

- **The Multiplicative and Combined Multiplex PageRank** are able to extract relevant information not reducible to the PageRank in single networks.

Multiplex PageRank collaboration

A. Halu, R. J. Mondragon, P. Panzarasa and G. Bianconi

PLoS ONE 8(10) e78293 (2013).

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