Information slows down hierarchy growth

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MOTIVATION

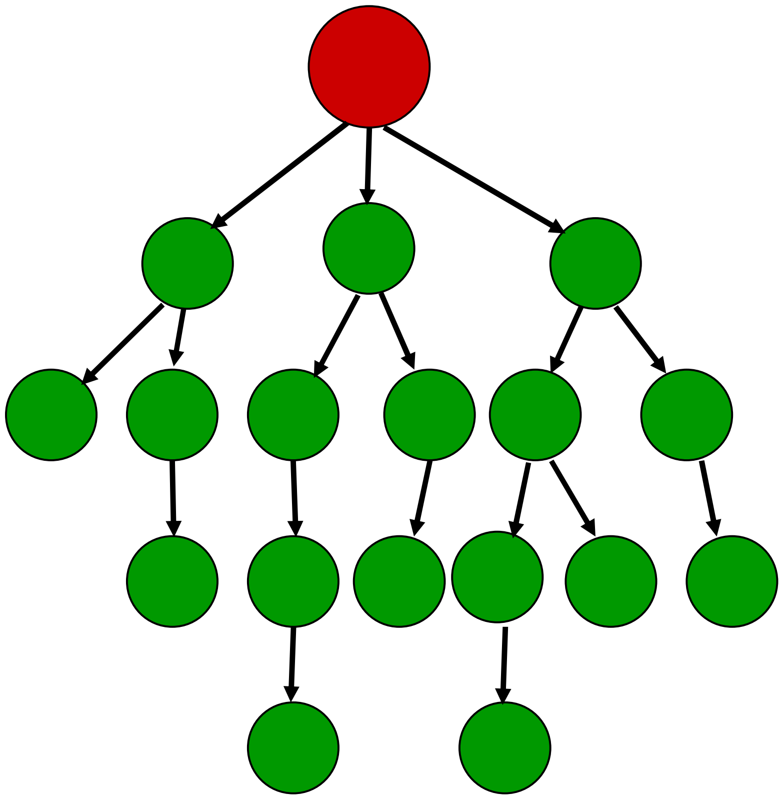
In growing social groups, new individuals try to occupy the best place in the existing social hierarchy. They know only **a limited fraction** of society, and can make their choice based only on this **limited knowledge**.

THE AIM OF THE STUDY

Check how the **information availability** influences network topology or more precisely, to investigate the emergence of consecutive **hierarchy levels**.

The concept of hierarchy in complex networks, a few examples...

- the position in directed networks where arcs define ”higher-lower” relations between the nodes

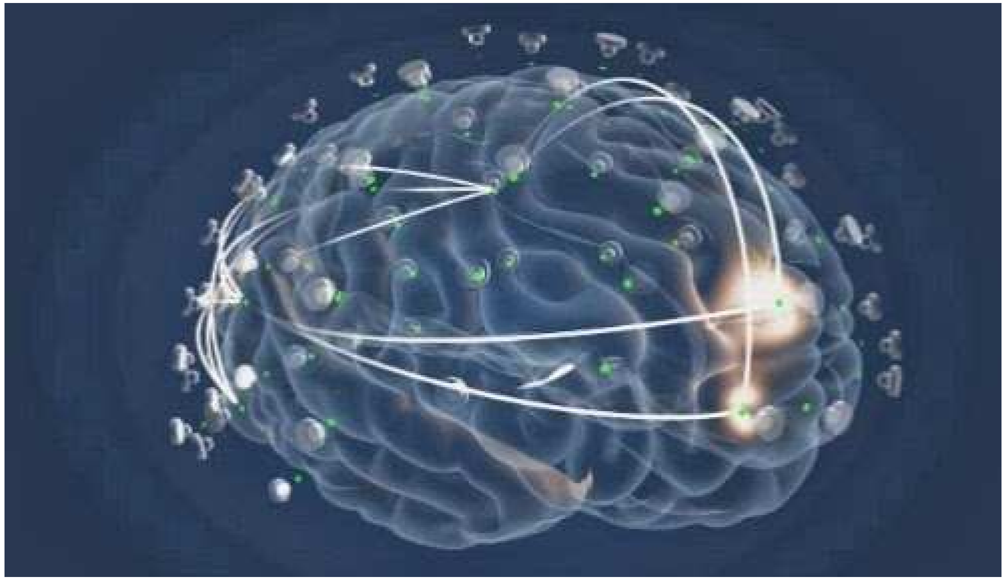


The concept of hierarchy in complex networks, a few examples...

- the importance and the role of a node in a social community structure

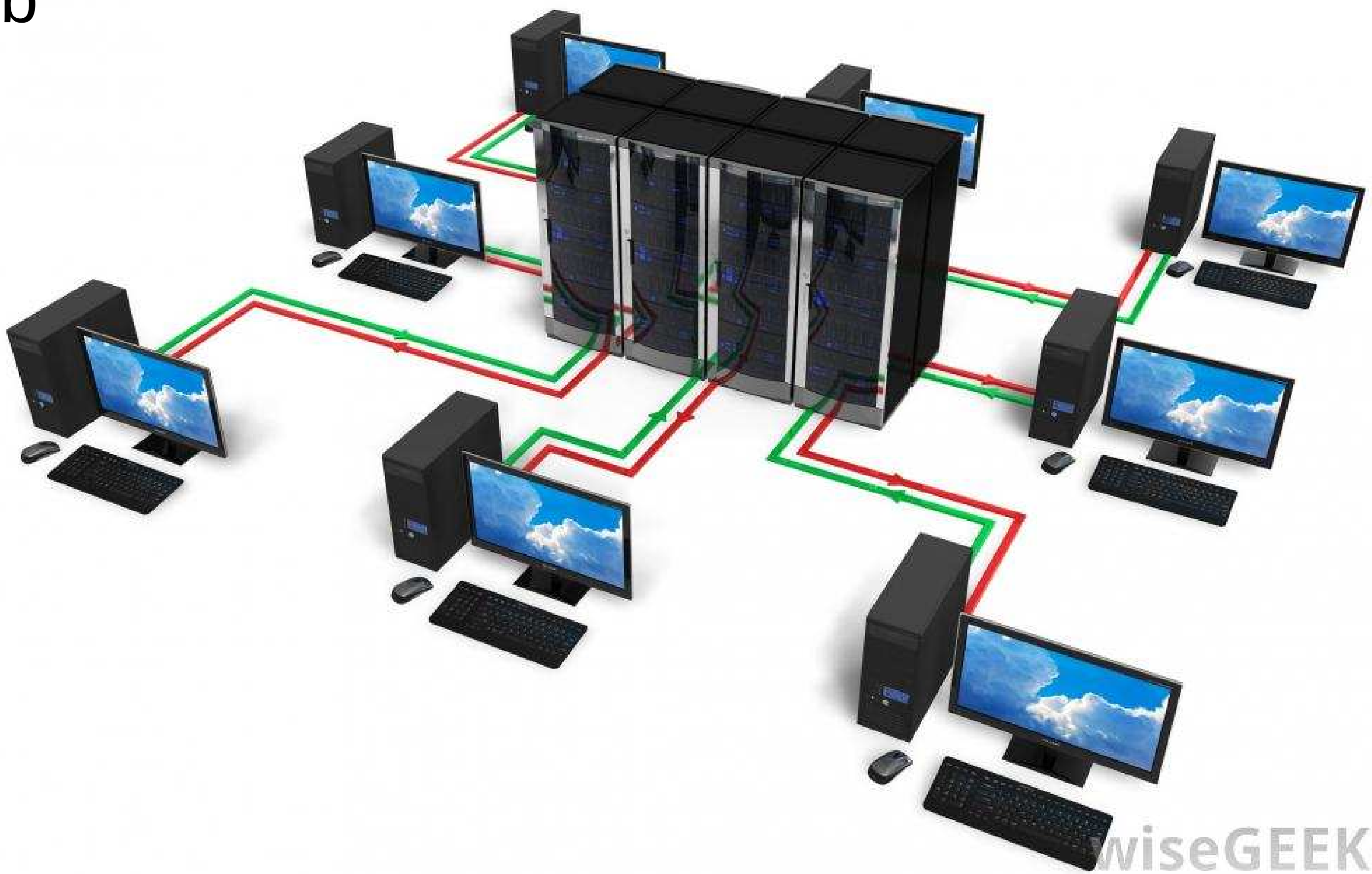
The concept of hierarchy in complex networks, a few examples...

- the participation of a node in activity patterns in neural networks



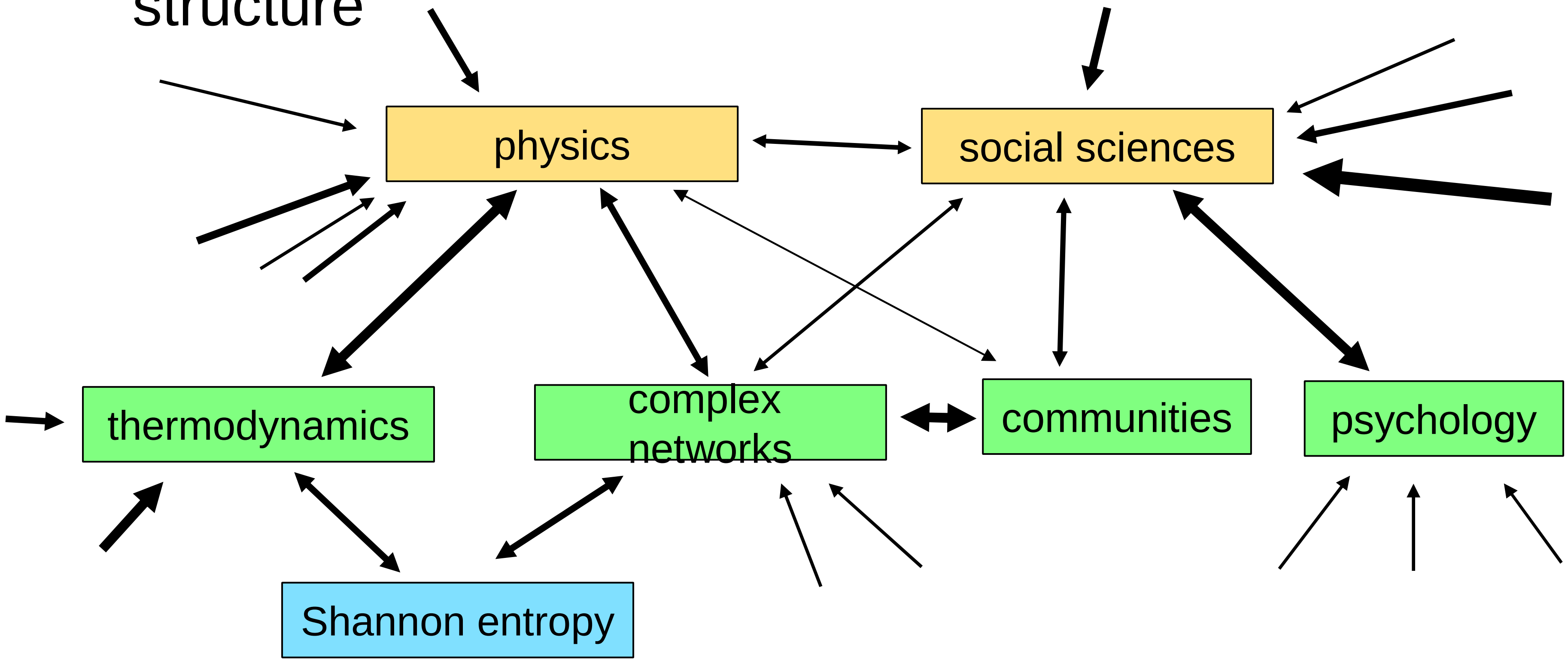
The concept of hierarchy in complex networks, a few examples...

- a node's importance as a potential communication hub



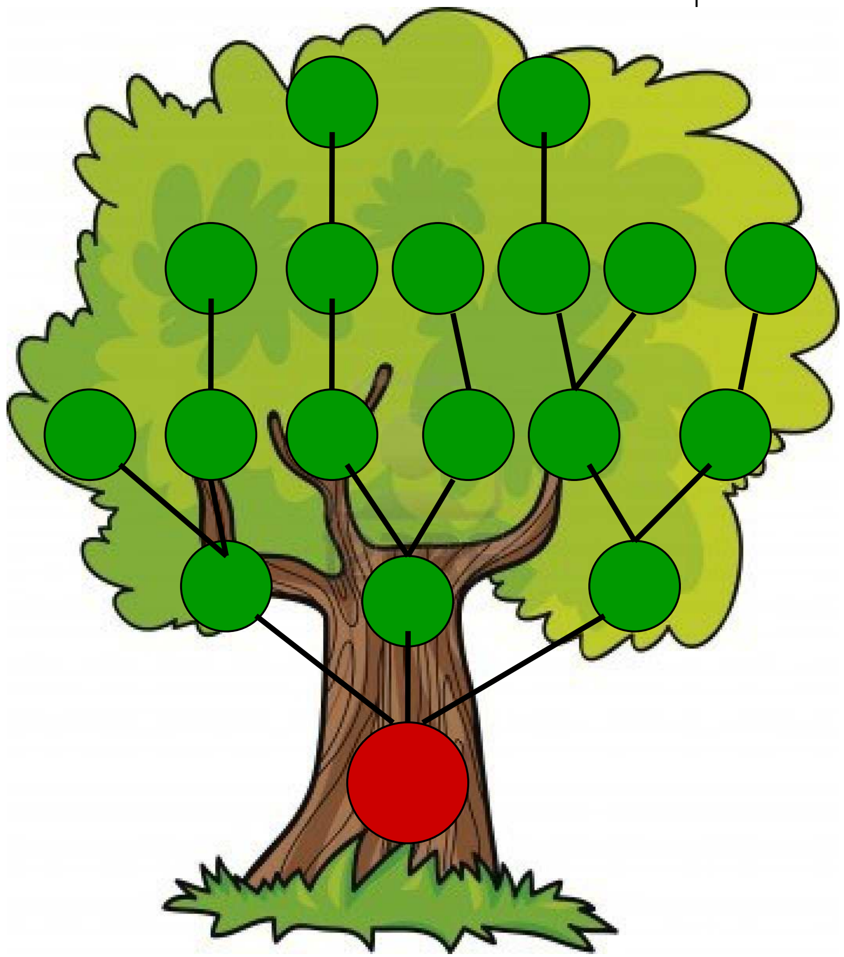
The concept of hierarchy in complex networks, a few examples...

- a node's relational importance in a knowledge structure



The tree graph

- a network generally agreed to be hierarchical
- natural root definition and natural hierarchy levels
- can be considered a ”perfect” hierarchy



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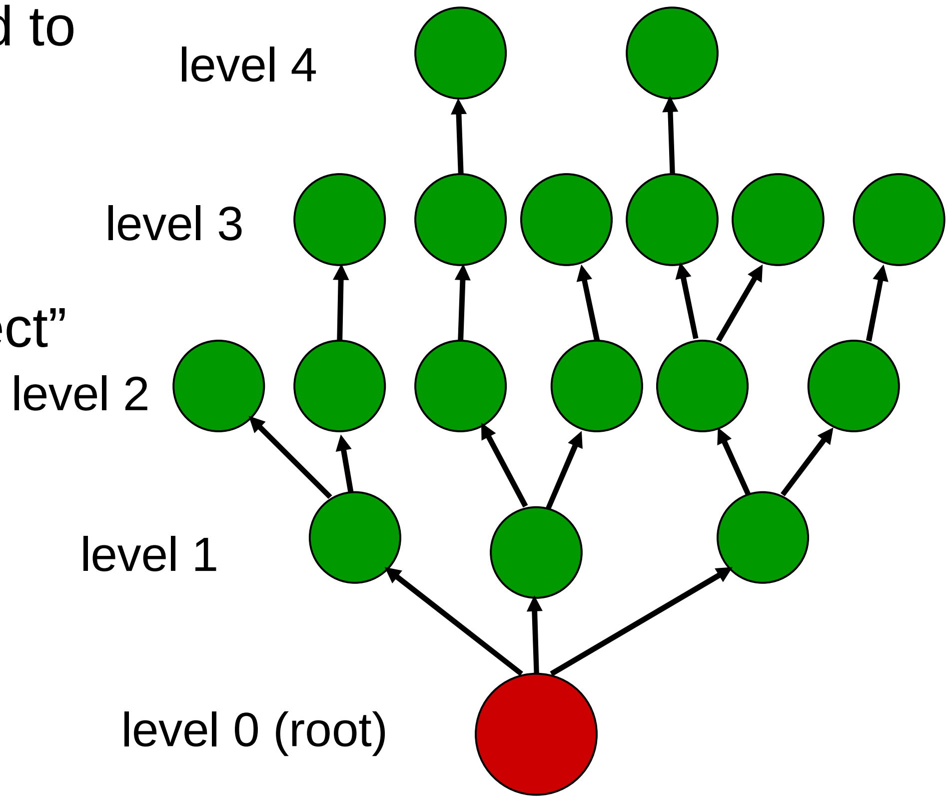

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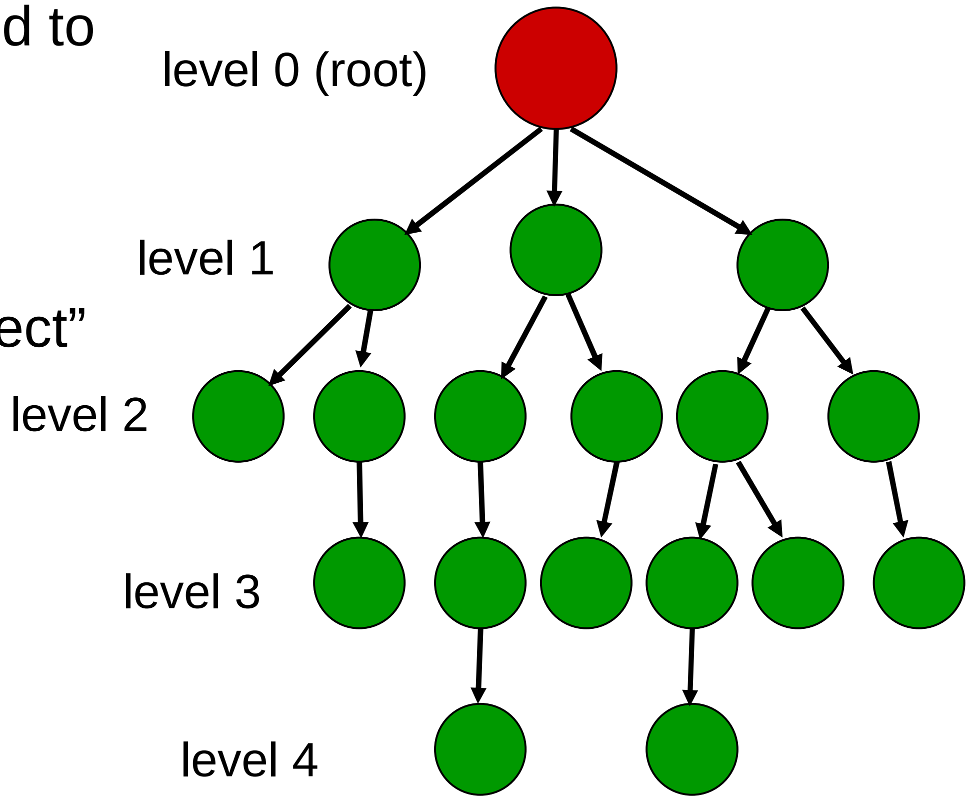


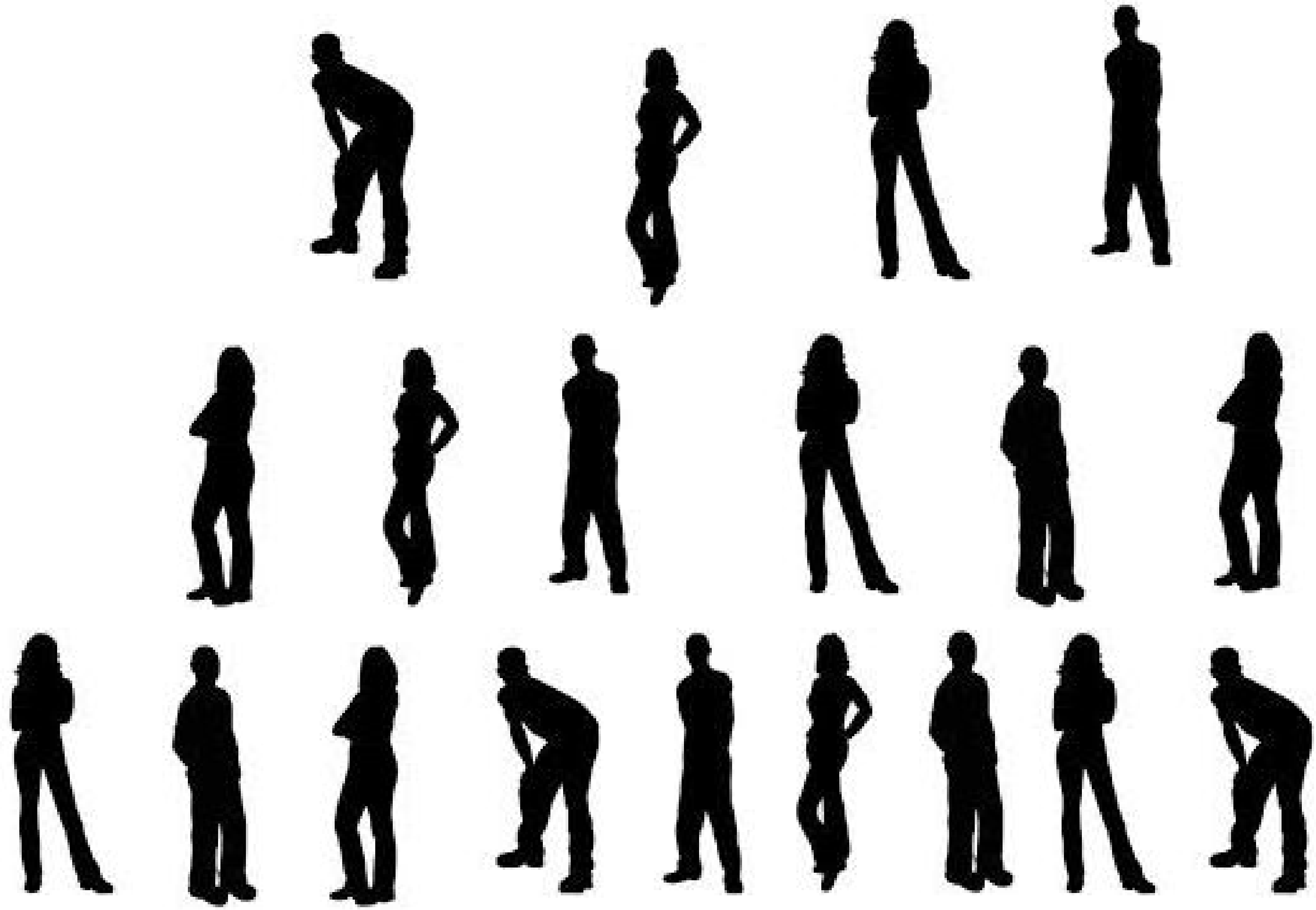
How does a hierarchical community expand ?

Tournament growing tree !

Tournament growing tree

At each time step we add a new node and choose one existing node to connect it to

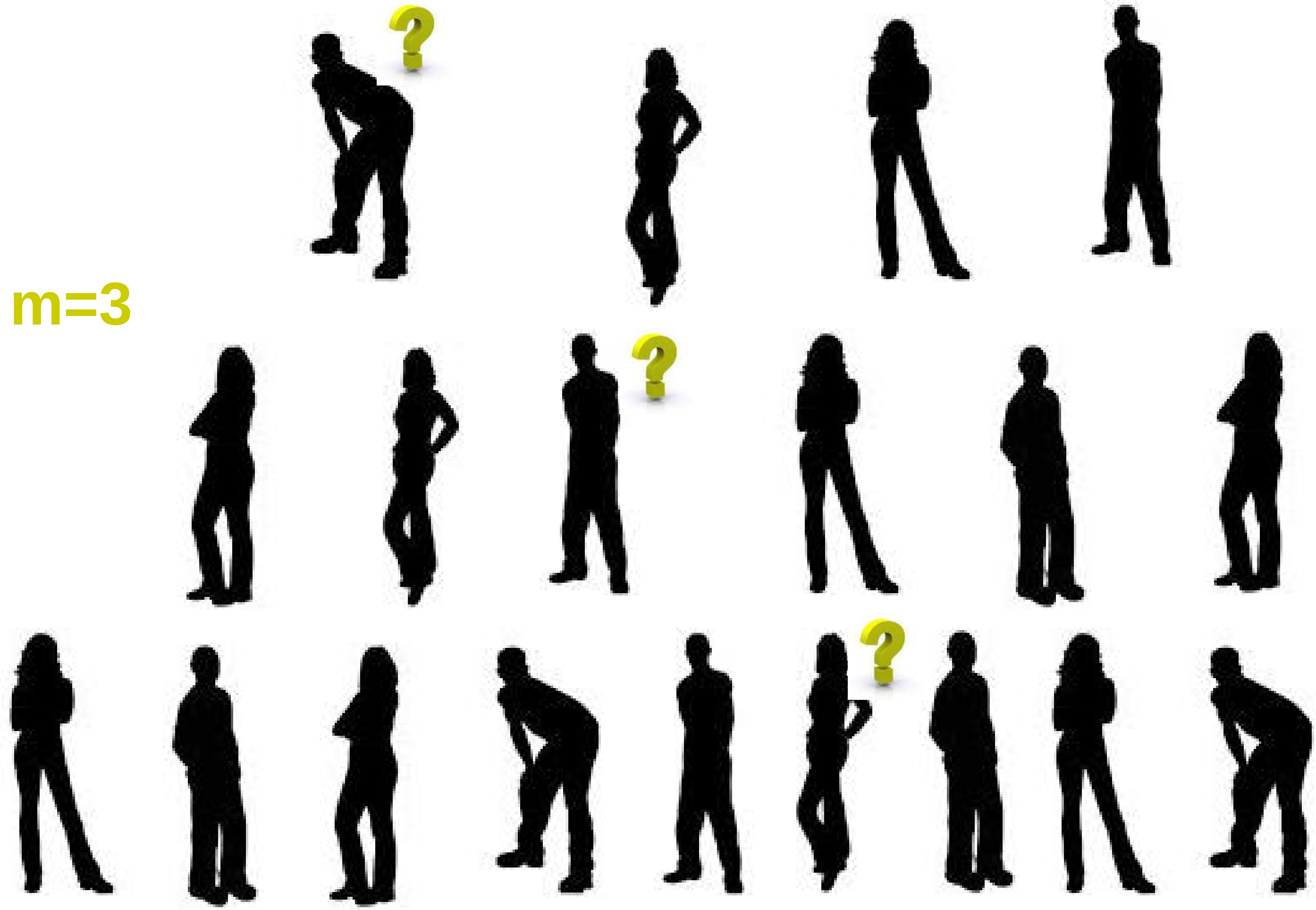
Community leader, tree root



The choice of a node to connect to is that of a ”tournament selection”:
Subset of ***m* random nodes is selected** from among the nodes already present

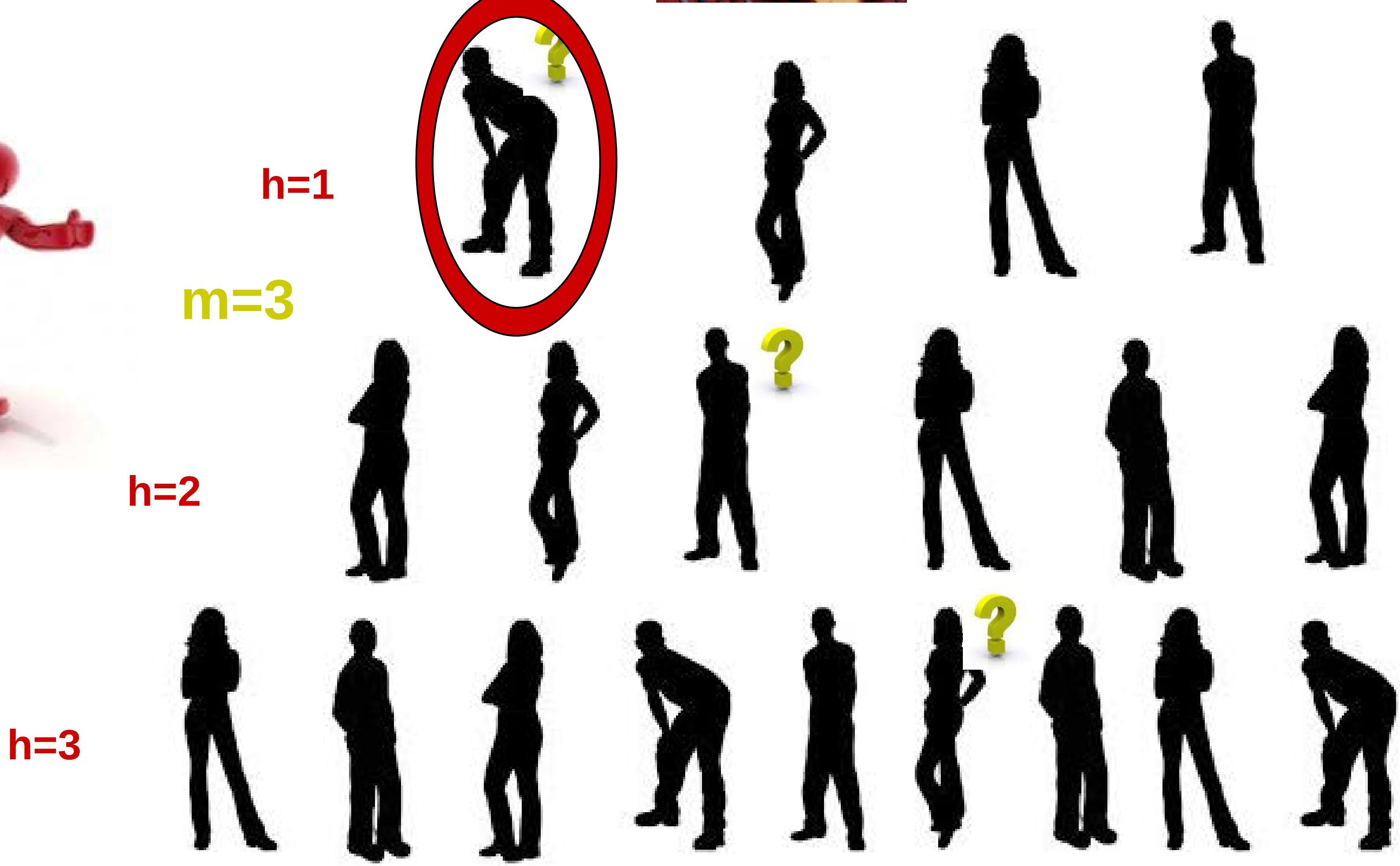
Community leader, tree root

m=3



A node at **the best hierarchy level** (**lowest h**) from among these is chosen to be connected to

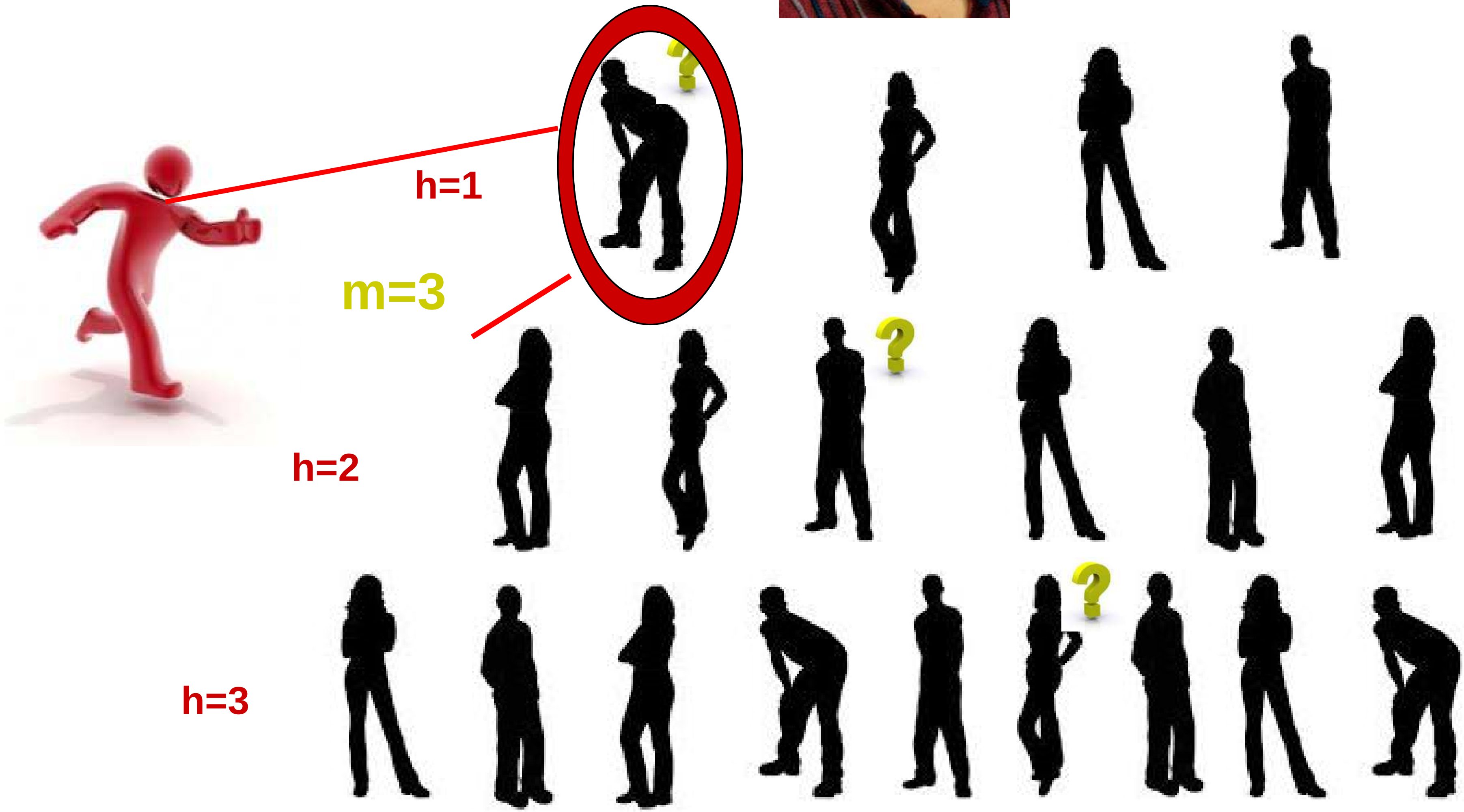
Community leader, tree root



A node at **the best hierarchy level** (**lowest h**) from among these is chosen to be connected to

h=0

Community leader, tree root



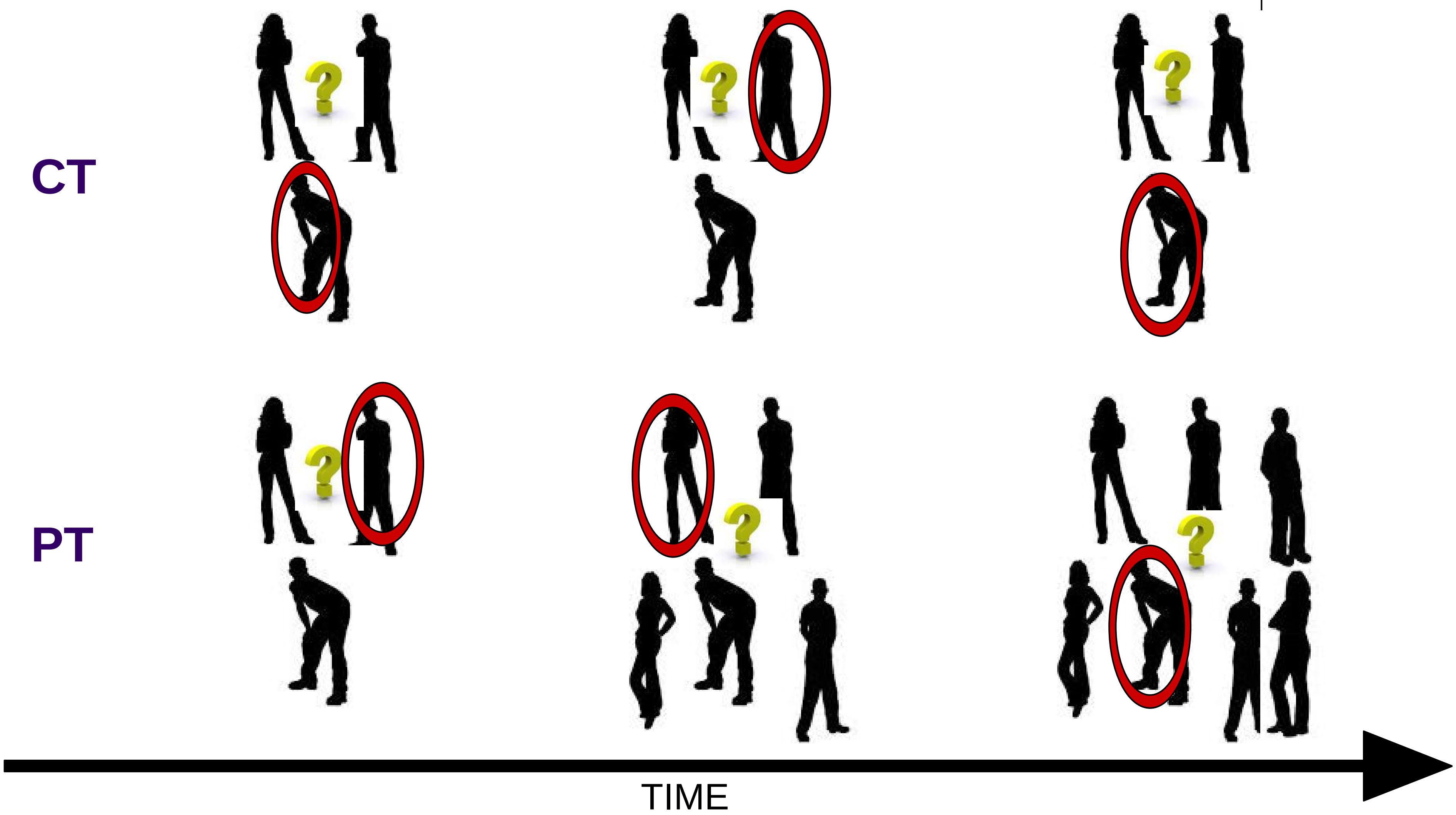
How does a hierarchical community expand ?

- Limited size of a tournament reflects limited availability of information
- Two tournament variants :

  -model with a given *constant tournament size,* m=const, CT model

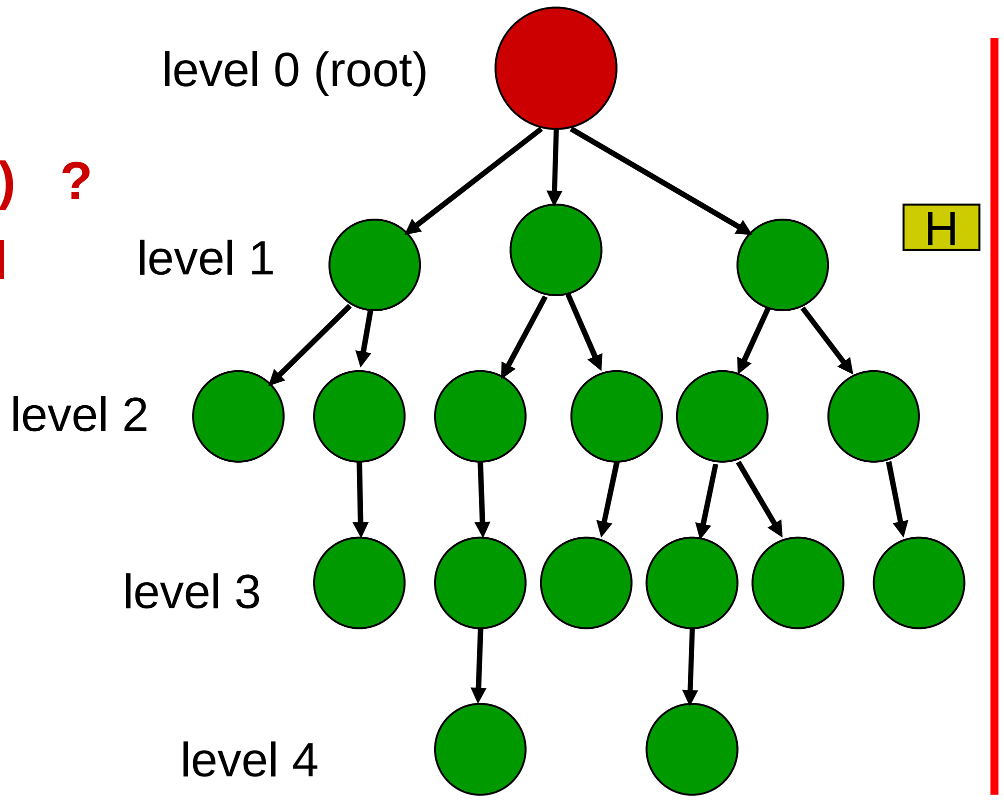
  - a model with random, *proportional tournament size,* $m = \alpha t$, PT model (Poisson T).

TOURNAMENT SIZE



Questions

- **Level occupation N_h(t) ?**
- **birth times t_h ?**
- **Tree heigh h_{max}(t) = H(t)   ?**
- **Occupation of the last level N_h_{max}(t)=N_H(t) ?**
- **...**
- **....**



Number of nodes at hierarchy level h>0 in time (CT model)

$$\frac{dN_h(t)}{dt} = \frac{\binom{N(t)-\sum_{i=0}^{h-2} N_i(t)}{m} - \binom{N(t)-\sum_{i=0}^{h-1} N_i(t)}{m}}{\binom{N(t)}{m}}$$

$$N_0(0)=1,$$
$$N_{h>0}(0)=0$$

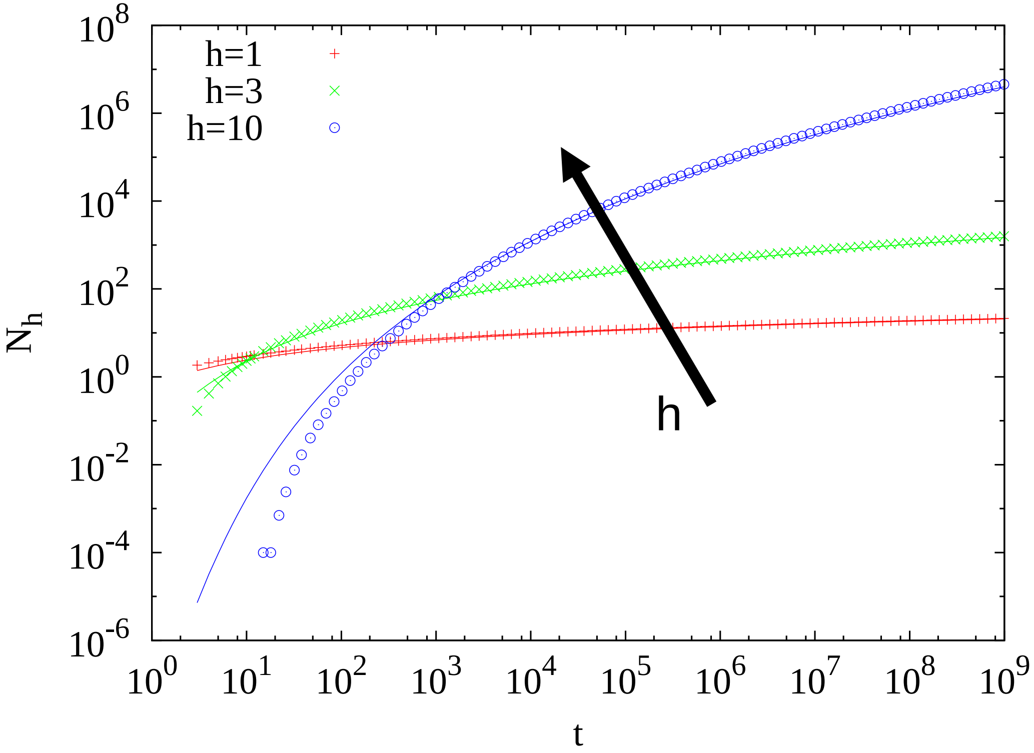
- For m=1 $\quad N_h(t) = \frac{1}{h!}\left(\ln(t+1)\right)^h$

- For m>1 $\quad N_1(t) = m\ln(t+1)$

$N_{h>1}(t)$ - numerical integration of the rate equation

New roots do not emerge, the number of nodes at level h = 0 is constant in time and equal to $N_0(t)=1$ (in both models).

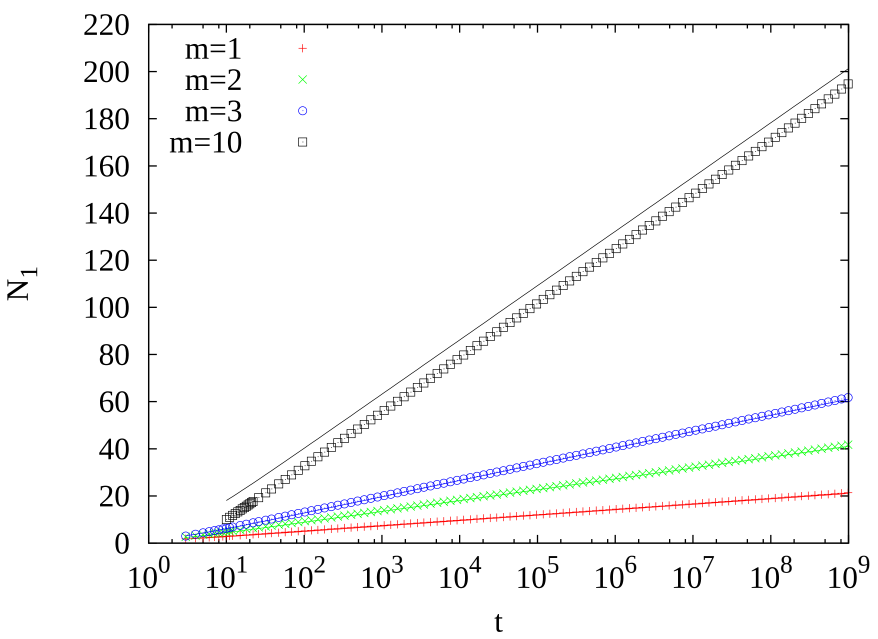
Number of nodes at hierarchy level h in time for m=1 (CT model)

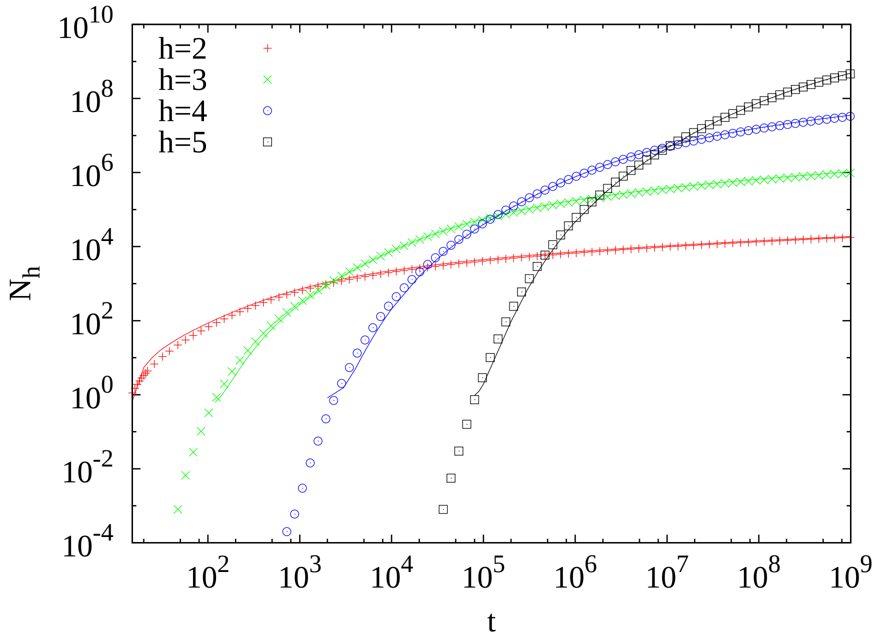


$$N_h(t) = \frac{1}{h!}\left(\ln(t+1)\right)^h$$

Number of nodes $N_h(t)$ at each hierarchy level h increases as a logarithm to power h.

Number of nodes at hierarchy level h in time for m>1 (CT model)





Logarithmic growth of number of nodes $N_1(t)$ at the first hierarchy level $h = 1$ for diferent sizes of the tournament $m = 1; 2; 3; 10$.

Hierarchy levels that were **born earlier grow slower** than the ones following them.

Number of nodes at hierarchy level h in time (PT model)

$$q = 1 - \alpha$$

$$\frac{dN_h}{dt} = \frac{\left(1 - q^{N_{h-1}}\right) \cdot \prod_{i=0}^{h-2} q^{N_i}}{1 - q^{t+1}}$$

For h>2 only numerical solution

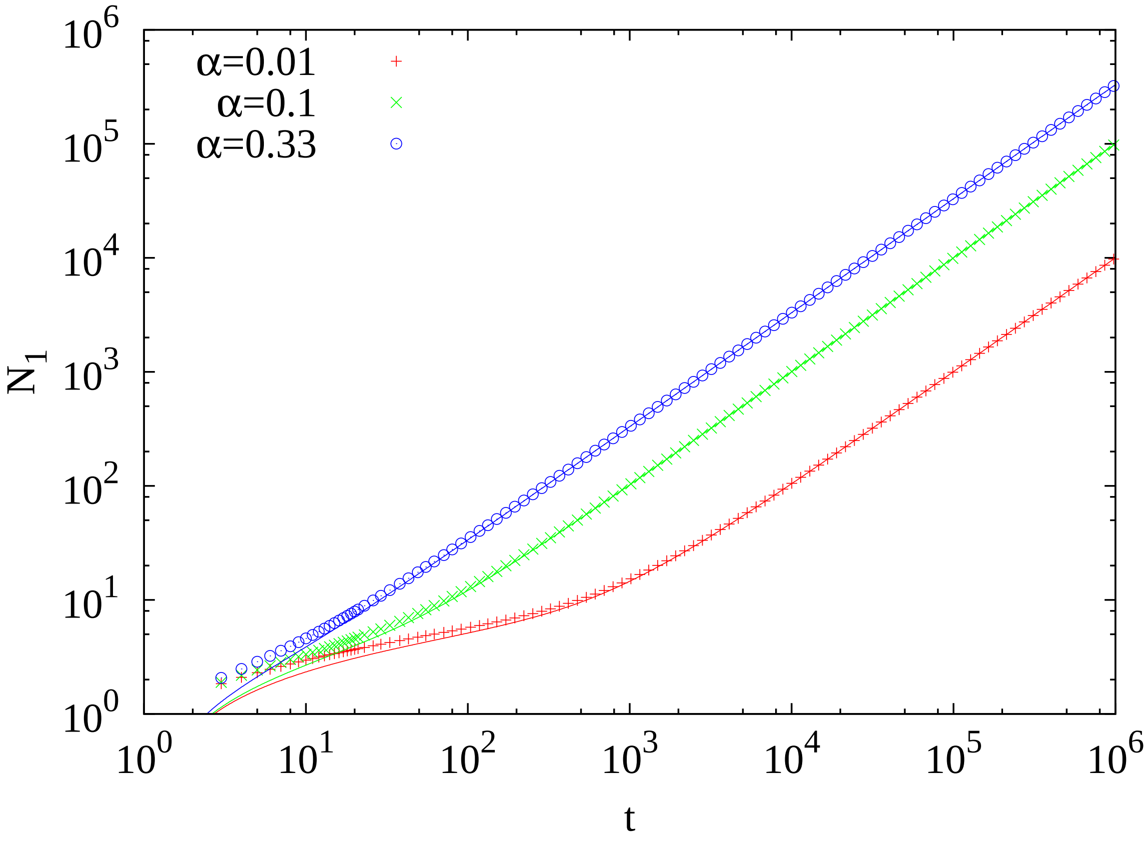
$$\frac{dN_1}{dt} = \frac{1-q}{1-q^{t+1}}$$

$$\text{for} \quad t >> 1 \quad N_1(t) \approx \alpha t \quad \text{and} \quad N_2(t) \approx qt$$

$$N_1(t) = (1-q)t + (1-q)\frac{\ln(1-q)}{\ln q} - (1-q)\frac{\ln\left(1-q^{t+1}\right)}{\ln q}$$

$$N_2(t) = qt + q\frac{\ln(1-q)}{\ln q} - q\frac{\ln\left(1-q^{t+1}\right)}{\ln q} - \frac{1}{\ln q}\left(\frac{q}{1-q}\right)^q \left(\left(\frac{1-q^{t+1}}{q^{t+1}}\right)^{-(1-q)} - \left(\frac{1-q}{q}\right)^{-(1-q)}\right)$$

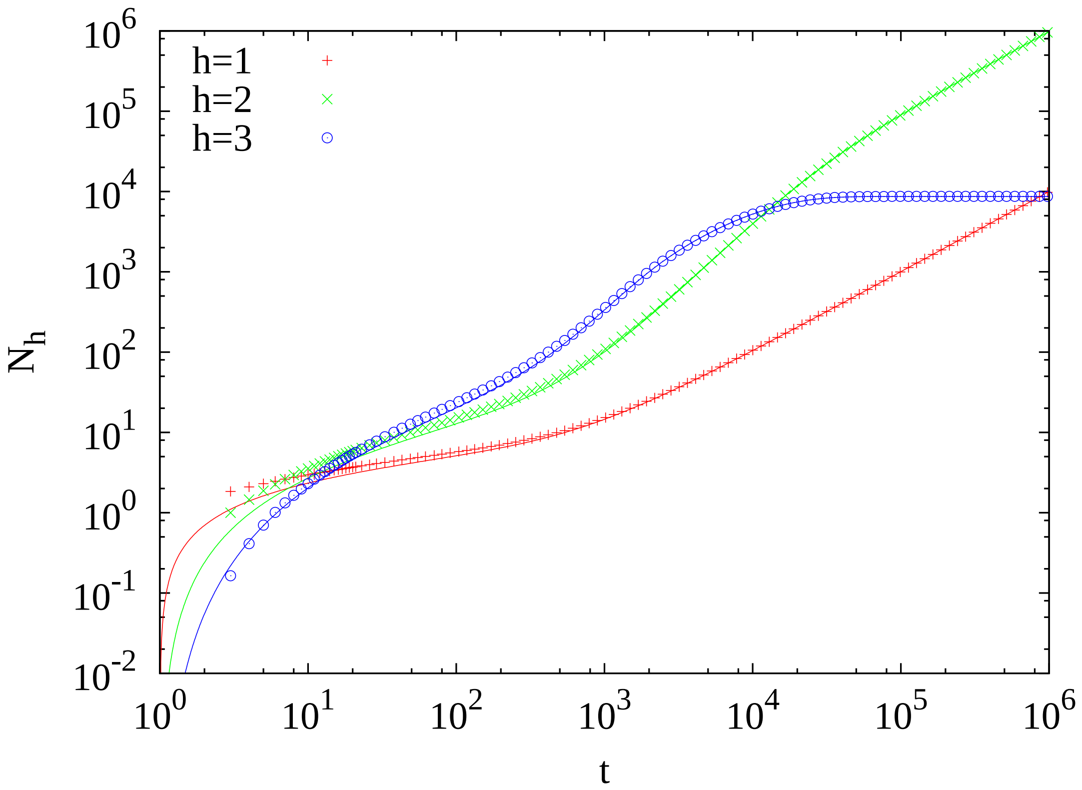
Number of nodes at first hierarchy level h=1 in time (PT model)



The hierarchy level directly below the root (h = 1) grows linearly with time.

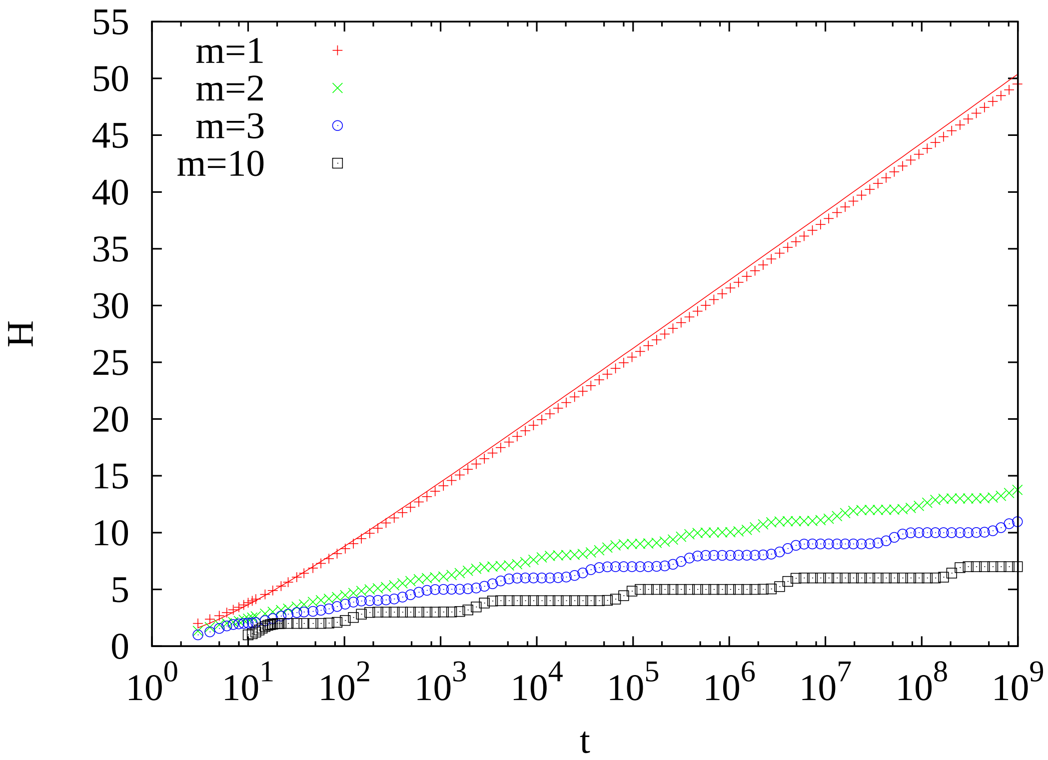
$$\text{for} \quad t >> 1 \quad N_1(t) \approx \alpha t$$

Number of nodes at hierarchy level h in time for α=0.01 (PT model)



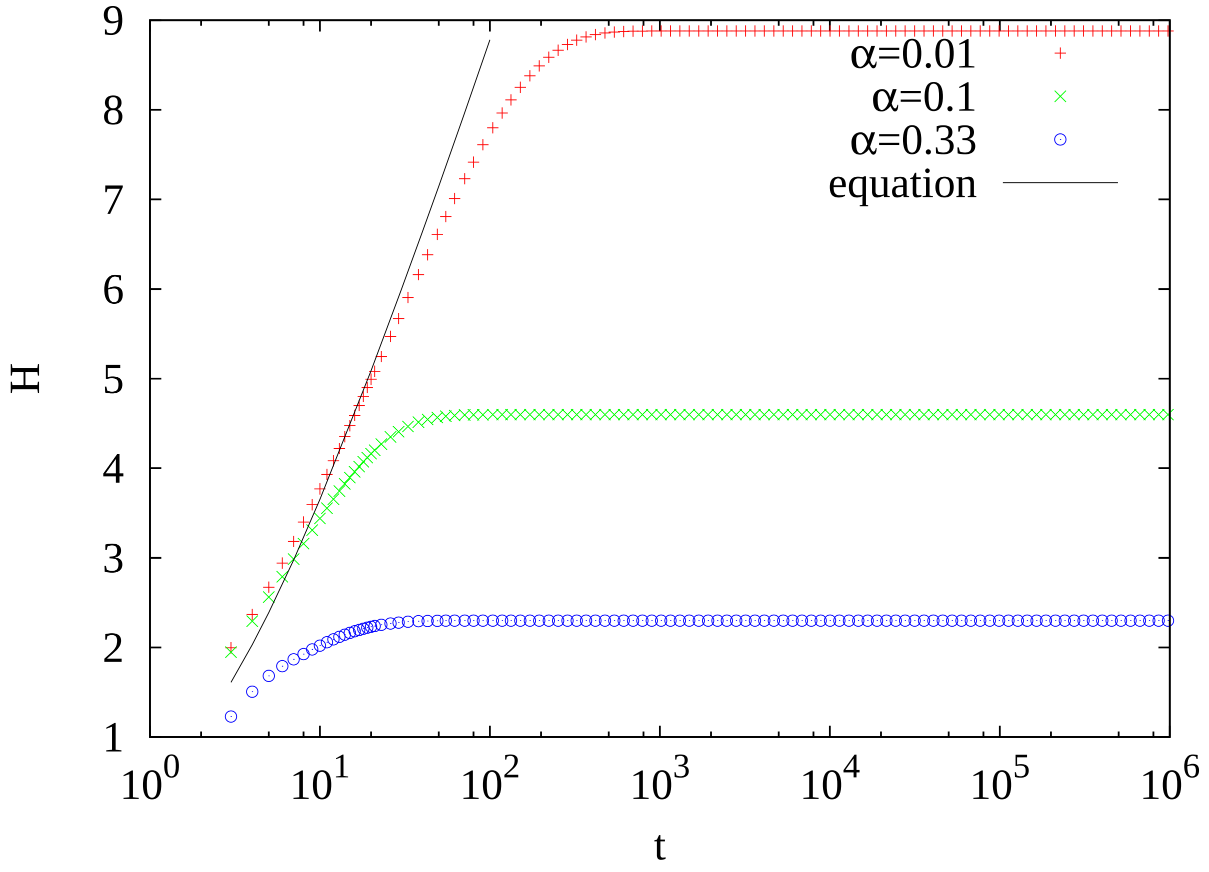
The growth of large trees is **monopolized by hierarchy levels** h = 1 and h = 2, since in the course of time levels h = 3 and worse stop growing.

Number of hierarchy levels in time (CT model)



Logarithmic growth of the number of hierarchy levels in time H(t) for m = 1 $H_{large}(t) \approx e\ln(t+1)$ and steplike growth for m > 1.

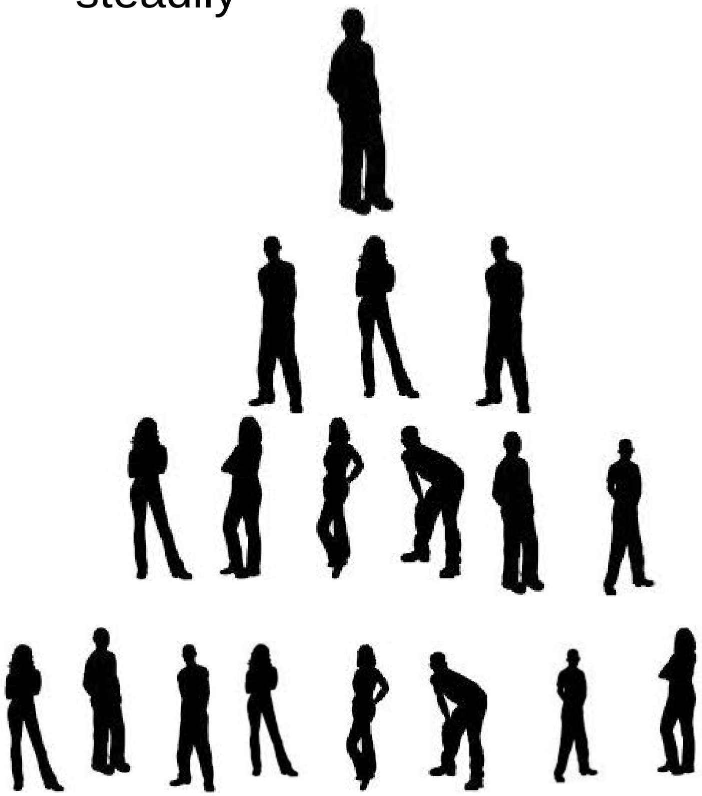
Number of hierarchy levels in time (PT model)



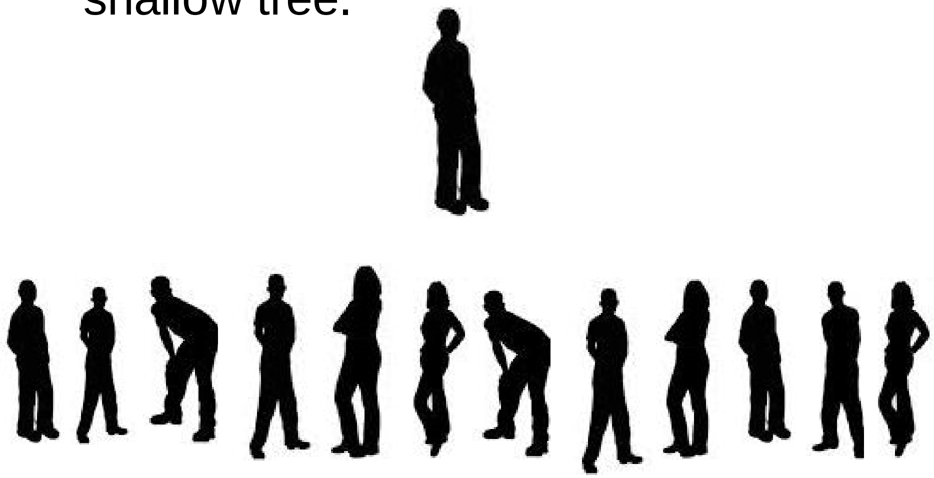
Number of hierarchy levels H(t) grows during initial time, and then saturates as last two levels monopolize the growth.

Conclusions

- **CT**: If new nodes know about a constant number of existing nodes, then the system grows steadily



- **PT**: If new nodes know about a fixed fraction of existing nodes, then the system dynamics changes in time and hierarchy growth slows down to a complete standstill, producing very wide and shallow tree.



Conclusions

- In a tree growth where nodes attach to the best known place in hierarchy, the availability of information restrains the emergence of hierarchy levels.
- The non-trivial observation is that it is the absolute amount of information, not relative, that governs this behavior.
- This is because information about only one well positioned node is required for the new node to connect well, regardless of how many nodes there are in total.
- A. Czaplicka, K. Suchecki, B. Minano, M.Trais, JAH, arXiv:1311.4460