Information slows down hierarchy growth

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Abstract

The introduction of Barabasi-Albert (BA) model offered a microscopic explanation of scale-free degree distribution emergence in a broad class of evolving networks through the mechanism of preferential attachment. This mechanism requires full information about the values of the relevant node attribute in the whole network. Such information is usually infeasible, if a larger real system is considered and therefore the question of the impact of the information limit – a limit of the amount of information new nodes possess about the whole system, is highly relevant. We have considered a growth of a tree graph, with new nodes attaching to existing ones taking into account their hierarchy levels. Motivated by observations of growing social groups, we assume that new nodes representing individuals will try to occupy the best places in the existing social hierarchy. Information constrains are modeled by a limited and random set of old nodes that they can connect to, in effect connecting to best-positioned node out of known ones. The aim of this study is to check how this limit influences the network topology, or more exactly the emergence of consecutive hierarchy levels. We treat the attachment process as a "tournament" that has a random set of participants, and the best node (the one on best hierarchy level) "wins" and is attached to. Two cases are considered, the constant tournament (CT) model where the size of the tournament group m is constant during the tree evolution and the proportional tournament (PT) model where it is growing proportionaly to the actual tree size (with coefficient α, forced at least 1 participant). We develop an analytical approach based on rate equations and perform numerical simulations of the growth process to test them. We find that the analytical results fit well to numerical simulations for both models, despite some quantitative-only disagreements. In the CT model, all hierarchy levels emerge in the tree but the birth time of the hierarchy level increases exponentialy or faster with its number. In the PT model, occupations of the first two hierarchy levels increase linearly, while worse levels do not grow at all, only appearing by chance during an early evolution stage. We conclude that in our model of tree growth where nodes attach to the best known place in hierarchy, the availability of information restrains the emergence of hierarchy levels. The non-trivial

observation is that it is the absolute amount of information, not relative, that governs this behavior. If new nodes know about a constant number of existing nodes, then the system grows steadily, as in the CT model (see left Figure). If they know about a fixed fraction of existing nodes, then the system dynamics change in time and hierarchy growth slows down to a complete standstill, as in the PT model (right Figure). This is because information about only one good positioned node is required to connect well yourself, regardless of how many nodes there are in total. Repeated connections to nodes at good hierarchy levels make it even easier to connect well, producing very wide and shallow tree.

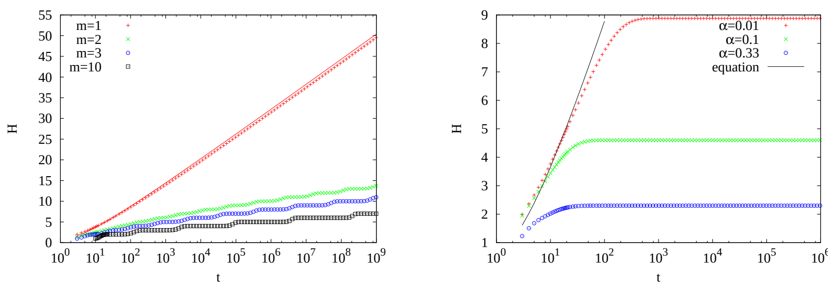

Figure 1: Growth of the total hierarchy depth/height in time in constant tournament (CT) model (left) and proportional tournament (PT) model (right). The logarithmic growth of number of hierarchy levels $H(t)$ for CT model depends on the tournament size m. The time before information stops hierarchy growth in PT model depends on tournament size growth speed α. Results of computer simulations are presented by symbols, analytical results by solid line. The "equation" is simple approximation of early PT model through CT model with $m = 1$.