Chaos at fifty: a statistical perspective

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Exactly fifty years ago Edward Lorenz published a paper with an unimpressive title in a journal outside the focus of the theoretical physicists' community [1]. The study remained unnoticed for years to past. Today, however, the occurrence of the Lorenz's paper is taken arguably as the birth of a science discipline nicknamed Chaos (or as a rebirth of a field originated by Henry Poincare 70 years or so earlier). At present, Chaos has been developed as both a conceptual basis and as methodology for a variety of natural and social nonlinear processes and phenomena. A great diversity of experimental results were obtained, which are at least in qualitative agreements with Chaos theory. The experiments span across: hydrodynamics and turbulence, geophysics, electronics, optics and laser physics, material sciences, chemistry, neurology, cognitive science, cardiology, etc.; see for example [2]. Chaos contributes also key ingredients to the modern methods of data processing and data characterization.

Since the chaotic system are inherently nonlinear, they could rarely by solved in a closed form. Hence, the efforts are concentrated on the interpretation and characterization of the numerical solutions. Toward this end a variety of methods are employed. Dynamical - computing the Lyapunov exponents, which measure the rate of exponential separation of two nearby chaotic orbits. Geometrical – computing the dimensions of the attractors, which as a rule are non-integer. A characteristic that bridges between the above two is called Kaplan-Yorke dimension. A perspective tools for characterization of the systems are based on the topology of the attractors [3].

In this talk we present recent results related to the statistical properties of simple 3D continuous chaotic systems. The core of examples for our study is provided by the family of chaotic systems discovered by Sprott [4]. In addition, we tackle cases with piecewise linear nonlinearities, (e. g. the Chua's circuit) as well as some specific newly discovered chaotic 3D systems. Our approach is based on classical single-point and two-point statistical functions, however, we compute them using a specially tailored for chaotic systems estimates and in particular estimates suitable for a very long data series. Such estimates should proof to be apt metrics for large data sets recorded in economics and social sciences. Finally, in a number of regimes determined by wide range of parameters values, we describe chaotic states using models based on approximate scaling at the onset of chaos [5].

References:

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