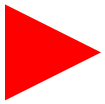
Knowledge, complexity, self-organization and social systems

Milan Rajković, Slobodan Maletić

Institute of Nuclear Sciences Vinča, Belgrade, Serbia

KNOWeSCAPE

QUANTUM GRAVITY
LOOP QUANTUM GRAVITY
STRING THEORY
QUANTUM SPIN NETWORKS



DISCRETE MODEL OF SPACE – SIMPLICIAL COMPLEX

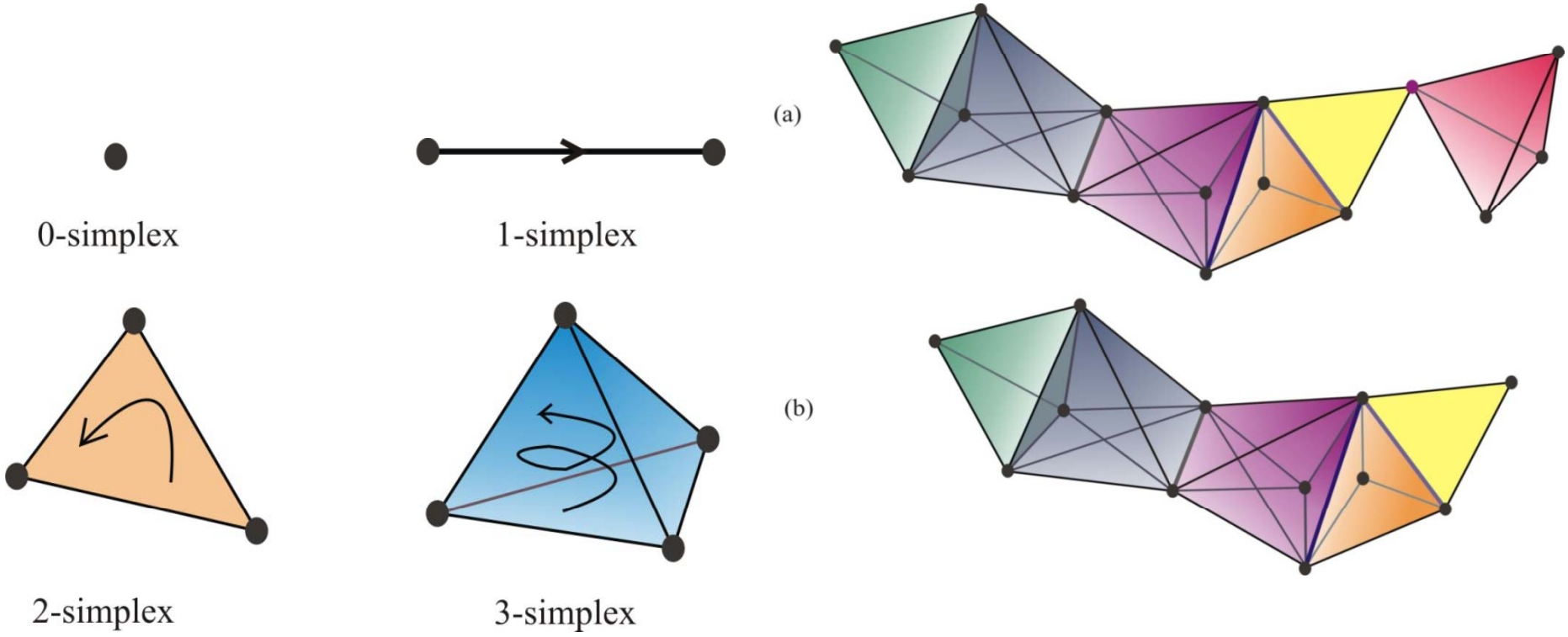
1. QM IS BASED ON NON-COMMUTATIVE QUANTITIES, AND SPACE COORDINATES ARE COMMUTATIVE

2. QM CALCULATIONS YIELD DIFFERENT ANSWERS WHICH GROW ARBITRARILY LARGE AS ONE CALCULATES QUANTITIES ON FINER AND SMALLER SCALES; DIFFICULTIES VANISH IF THERE IS A CUT-OFF.

3. SPACE-TIME METRIC IS NOT WELL DEFINED BECAUSE OF E.G. COLLAPSING SHELL OF MATTER CAN EVOLVE INTO A SINGULARITY.

A *Q*-DIMENSIONAL SIMPLEX OR *Q*-SIMPLEX IS AN (UN)ORDERED SET OF VERTICES:

$$\sigma_q = \langle v_1, v_2, \ldots, v_q, v_{q+1} \rangle$$



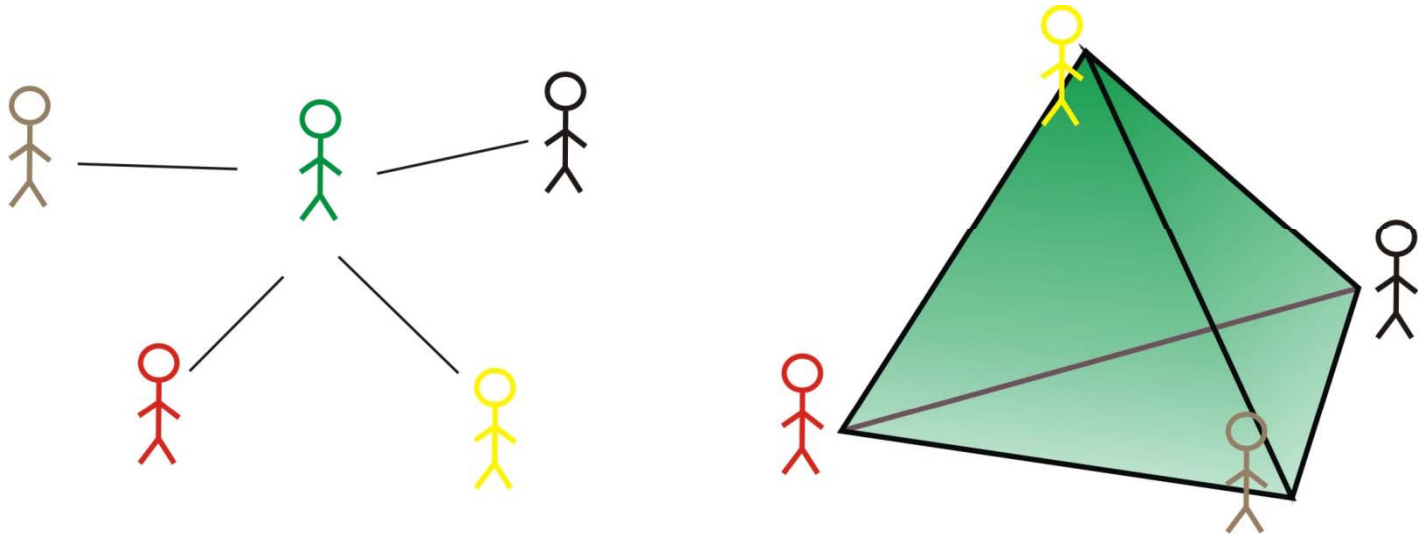
Why simplices and simplicial complex in complex (economic and social) systems?

1. Everyone can hear everyone else - complete graph on 4 vertices.

2. Each person may only whisper in the ear of another person. If each person can whisper to anyone else, this scenario is again modeled graph-theoretically by a complete graph on 4 vertices.

One-dimensional graph theory does not capture the distinction between a single 4-person conversation and six 2-person conversations.

Graphs capture pairwise relationships between collaborators but omit group collaboration information. For example, a paper co-authored jointly by 5 authors is treated the same as 5 separate papers each by a distinct pair of the 5 authors. In both cases, the result is a 5-clique.

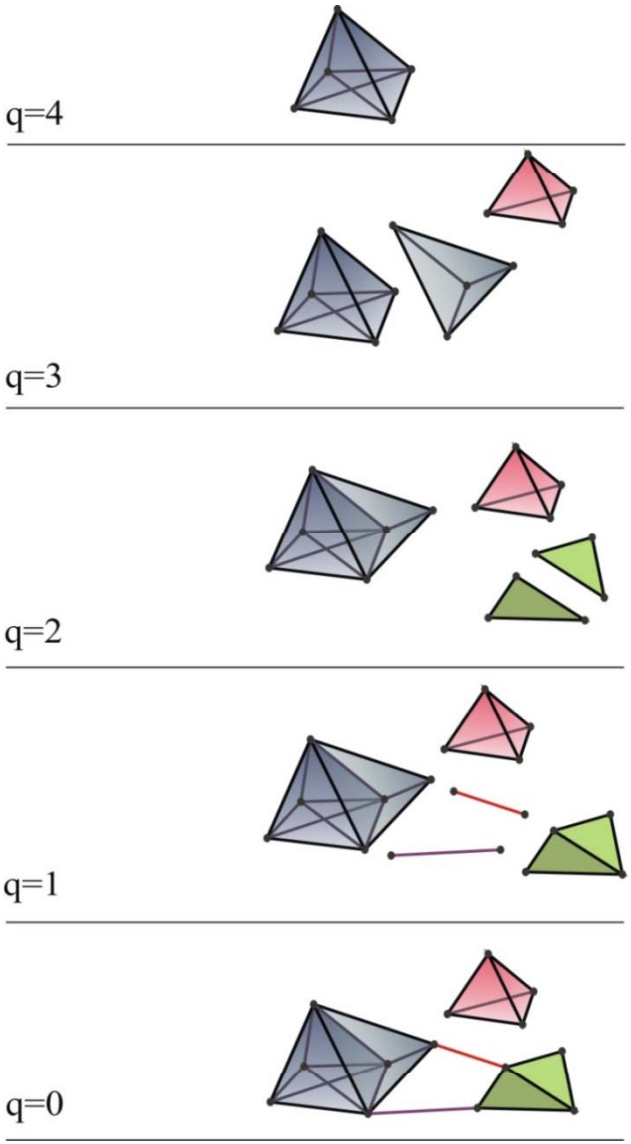


•Simplicial complexes are combinatorial versions of topological spaces - analyzed with combinatorial, topological and algebraic methods ...*Combinatorial Algebraic Topology*

Why simplices and simplicial complex in classical physics?

- **Discrete Differential Forms**
  - *Discrete Forms*
    - A discrete k-form on a piecewise-linear manifold assigns a real number to every oriented k-simplex of the manifold
      - *0-forms* are discrete versions of continuous scalar fields
      - *1-forms* are discrete versions of vector fields
    - *Exterior derivative* of a k-form ω is a (k+1)-form $d\omega$
    - A form ω is called *closed* if $d\omega = 0$

Q-VECTOR (STRUCTURE VECTOR)



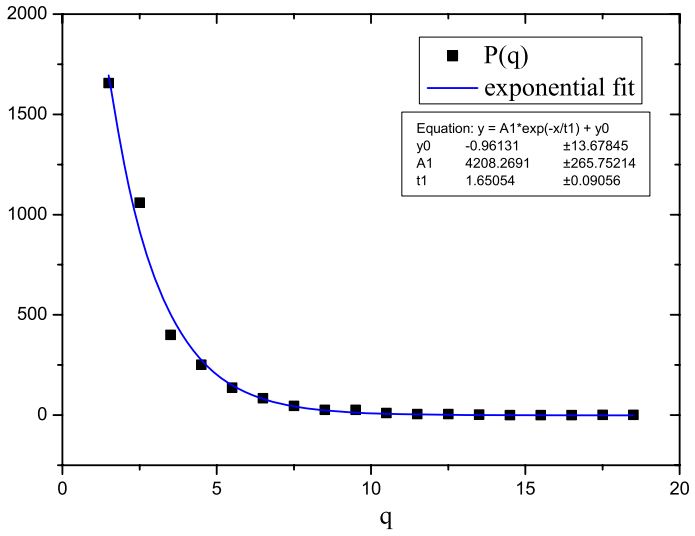
Q-CONNECTIVITY BETWEEN SIMPLICES IS AN EQUIVALENCE RELATION

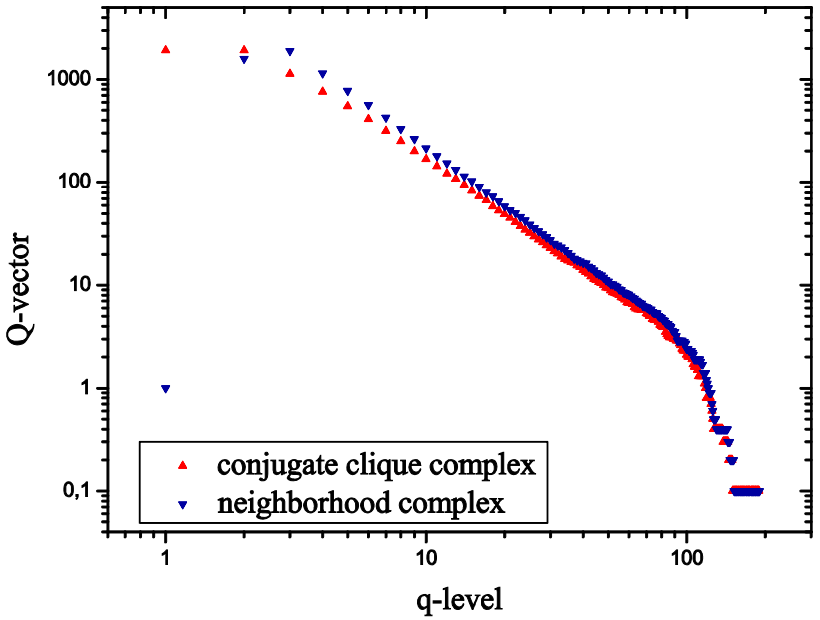
→ GENERATES THE PARTITION OF A SIMPLICIAL COMPLEX INTO *Q*-CONNECTED EQUIVALENCE CLASSES

THE NUMBER OF DISTINCT *Q*-CONNECTIVITY CLASSES IS STORED IN THE Q^{TH} ENTRY OF

Q-VECTOR (OR **STRUCTURE VECTOR**)

EXAMPLE 1: Q-VECTOR OF SCALE FREE NETWORK

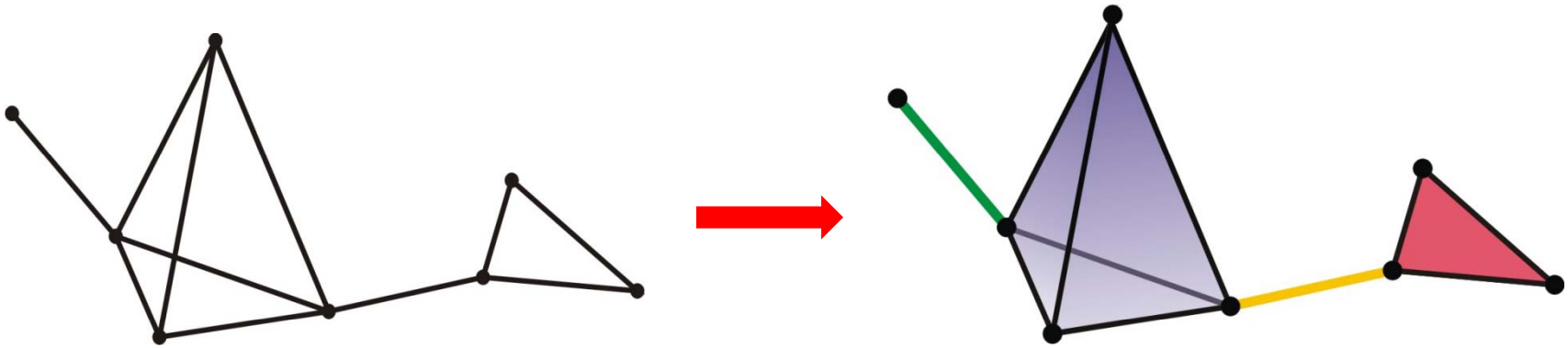




EXAMPLE 2: Q-VECTOR OF AN EXPONENTIAL NETWORK

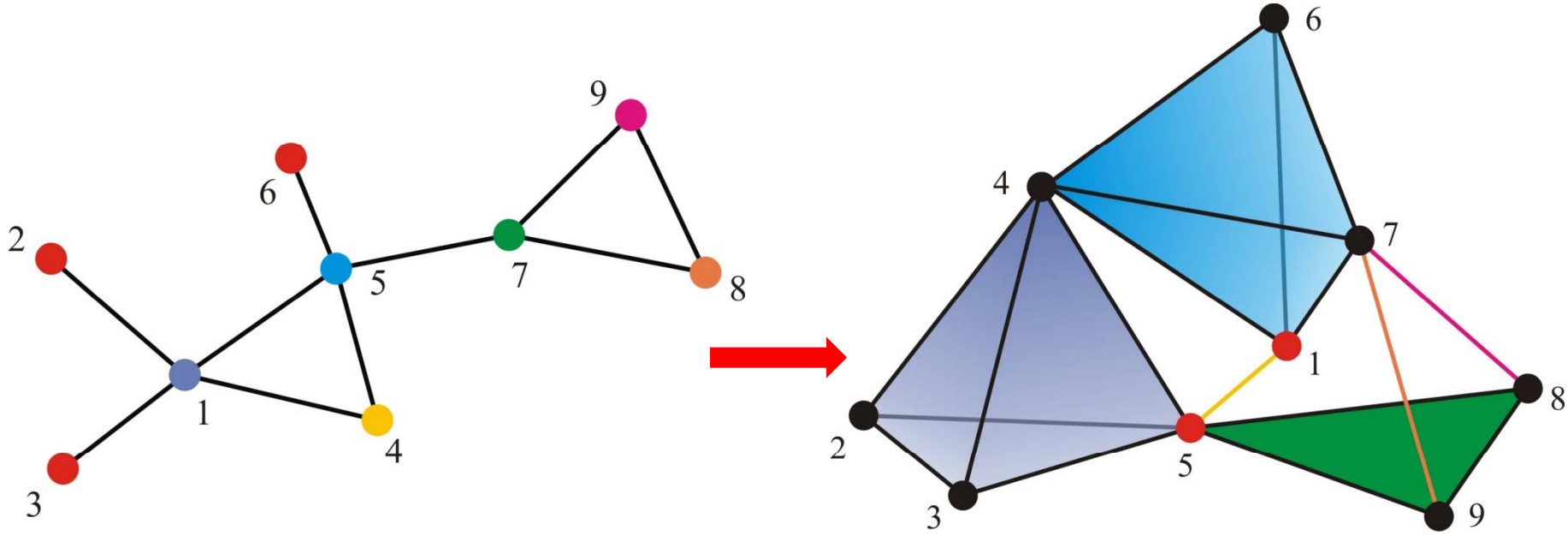
Graph → Clique complex

The vertices of clique complex are nodes of the underlying graph, and simplices are all maximal cliques (together with all their subcliques)

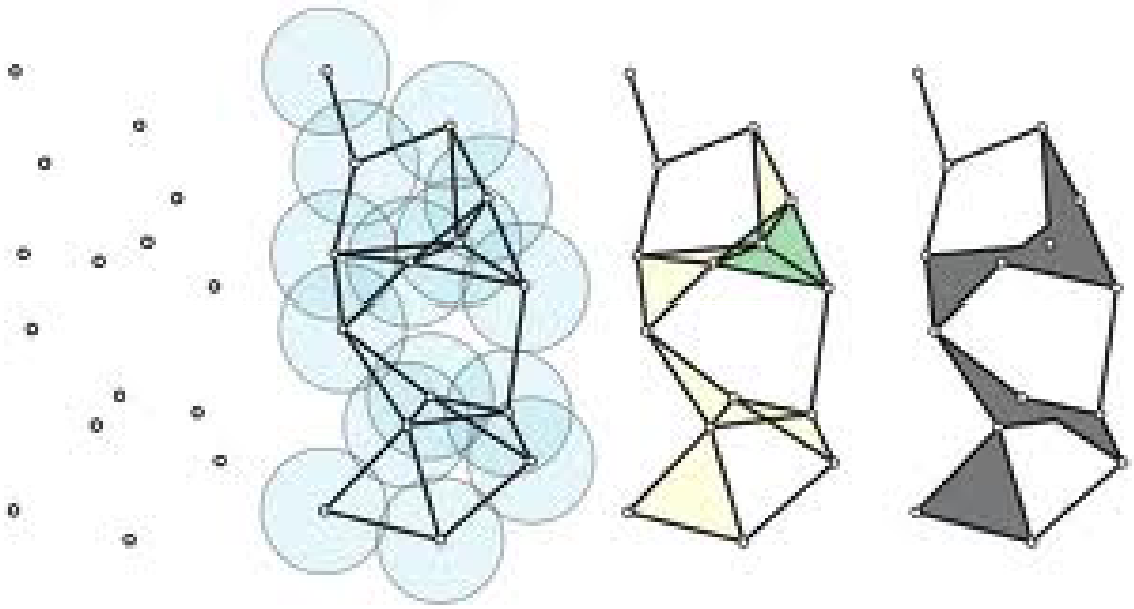


Graph → Neighborhood complex

The vertices are nodes of the underlying graph *G*, and to each node in *G* corresponds a simplex which contains all of its neighbors

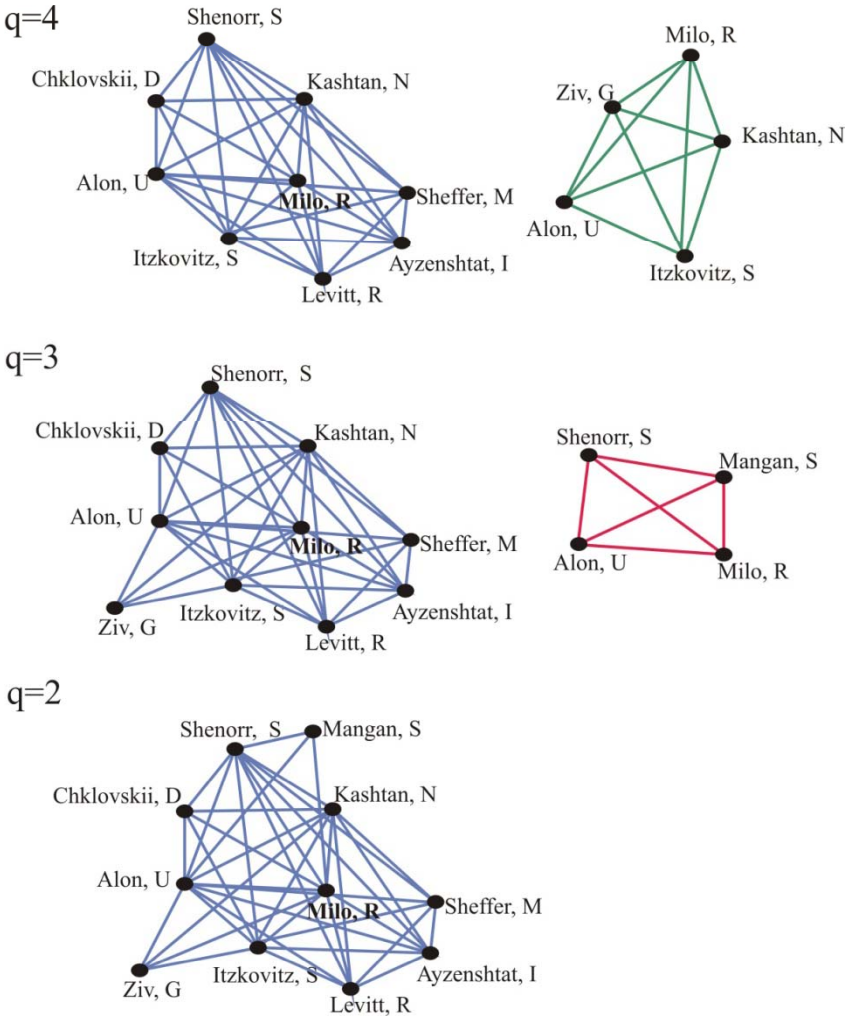


Chech complex; Rips complex ; Witness complex etc.

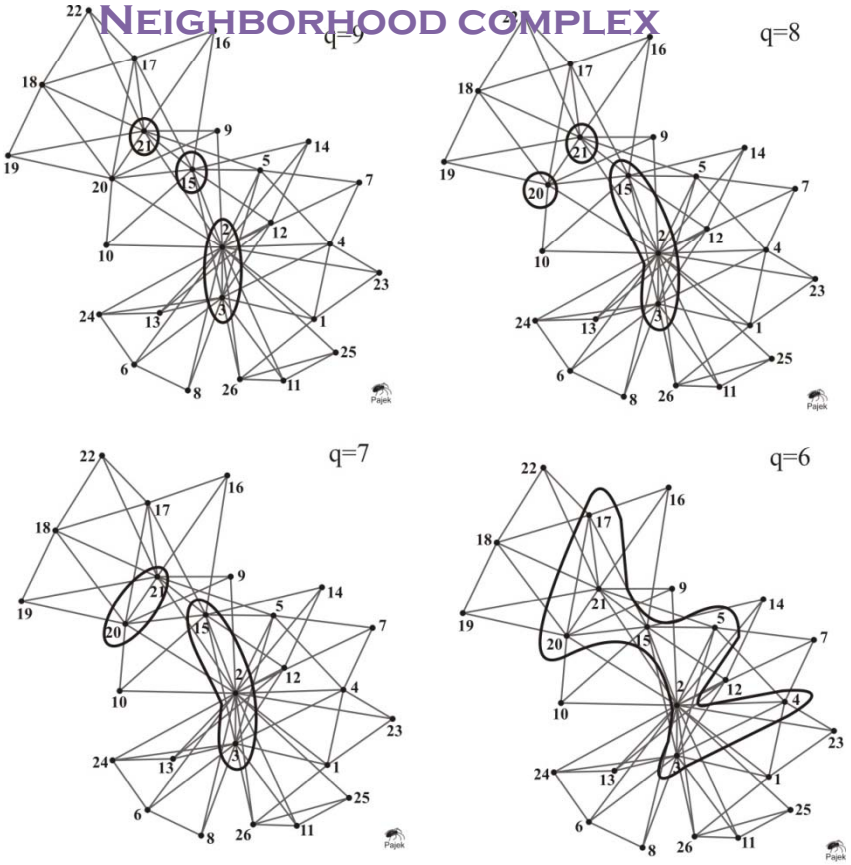


EXAMPLES OF CLIQUE AND NEIGHBORHOOD COMPLEX

COAUTHORSHIP NETWORK: CLIQUE COMPLEX



BRAIN NETWORK (NODES-BRAIN AREAS, LINKS-COMMUNICATION BETWEEN BRAIN AREAS): NEIGHBORHOOD COMPLEX



1-Caudal; 2-Anterior Cingulate; 3-Superior Frontal Gyrus; 4-Middle Frontal Gyrus; 5-Inferior Frontal Gyrus; 6-Medial Frontal Gyrus; 7-Insular Cortex; 8-Parahippocampnal Gyrus; 9-Superior Temporal Gyrus; 10-Inferior Temporal Gyrus; 11- Middle Temporal Gyrus; 12-Cingulate Gyrus; 13-Postcentral Gyrus; 14- Precentral Gyrus; 15-Inferior Parietal Lobule; 16-Superior Parietal Lobule; 17-Precuneus; 18-Cuneus; 19-Lingual Gyrus; 20-Fusiform Gyrus; 21-Inferior Occipital Gyrus; 22-Middle Occipital Gyrus; 23-Posterior Cingulate; 24-Unicate Fasciculus; 25-Lenticular Nucleus; 26-Thalamus

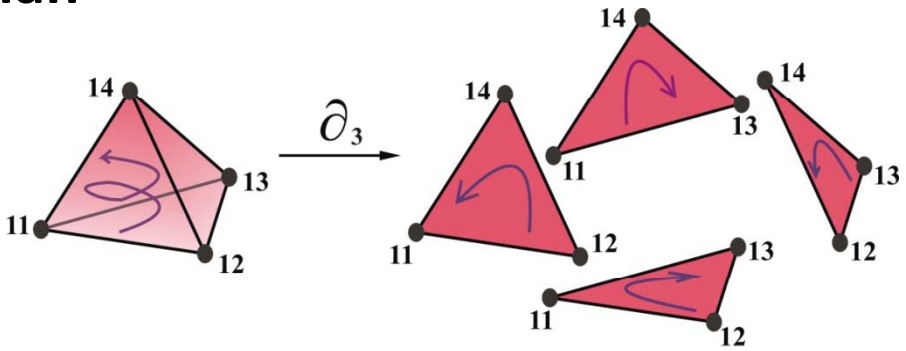
Homology & Combinatorial Laplacian

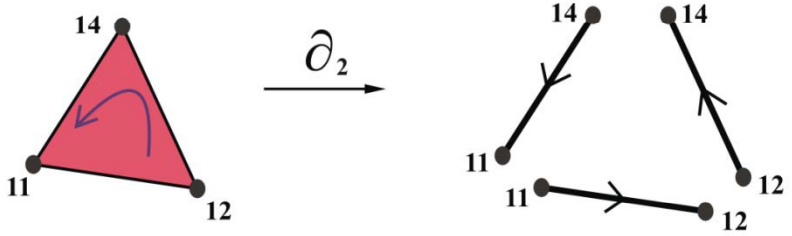
Chain group: C_q
Group whose bases is the set of all q-simplices of an oriented simplicial complex

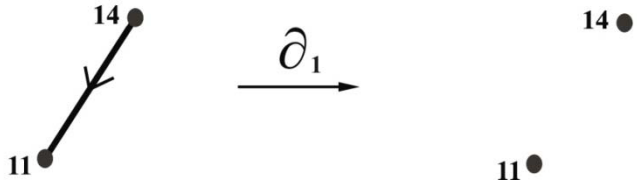
The elements are the linear combinations, called **chains**

Boundary operator on oriented simplices

$$\partial_q : C_q \to C_{q-1}$$







ADJOINT BOUNDARY OPERATOR $\partial_q^* : C_{q-1} \to C_q$

Q^{TH} COMBINATORIAL LAPLACIAN: $L_q : C_q \to C_q$

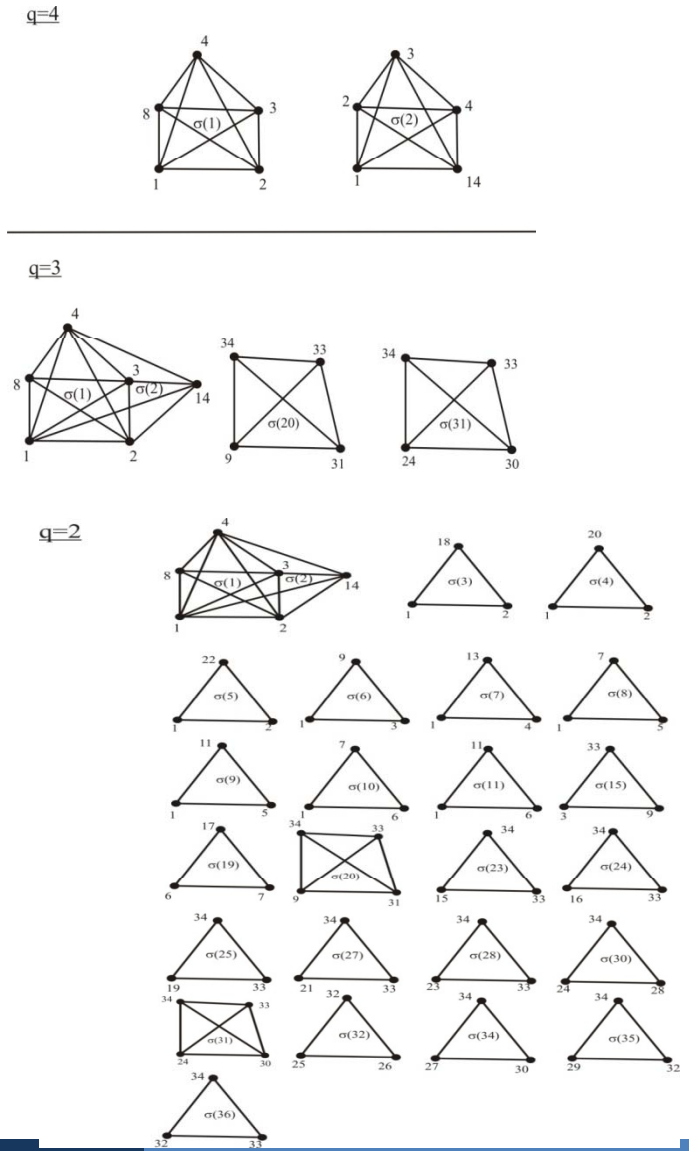
$$L_q = \partial_{q+1} \circ \partial_{q+1}^* + \partial_q^* \circ \partial_q$$

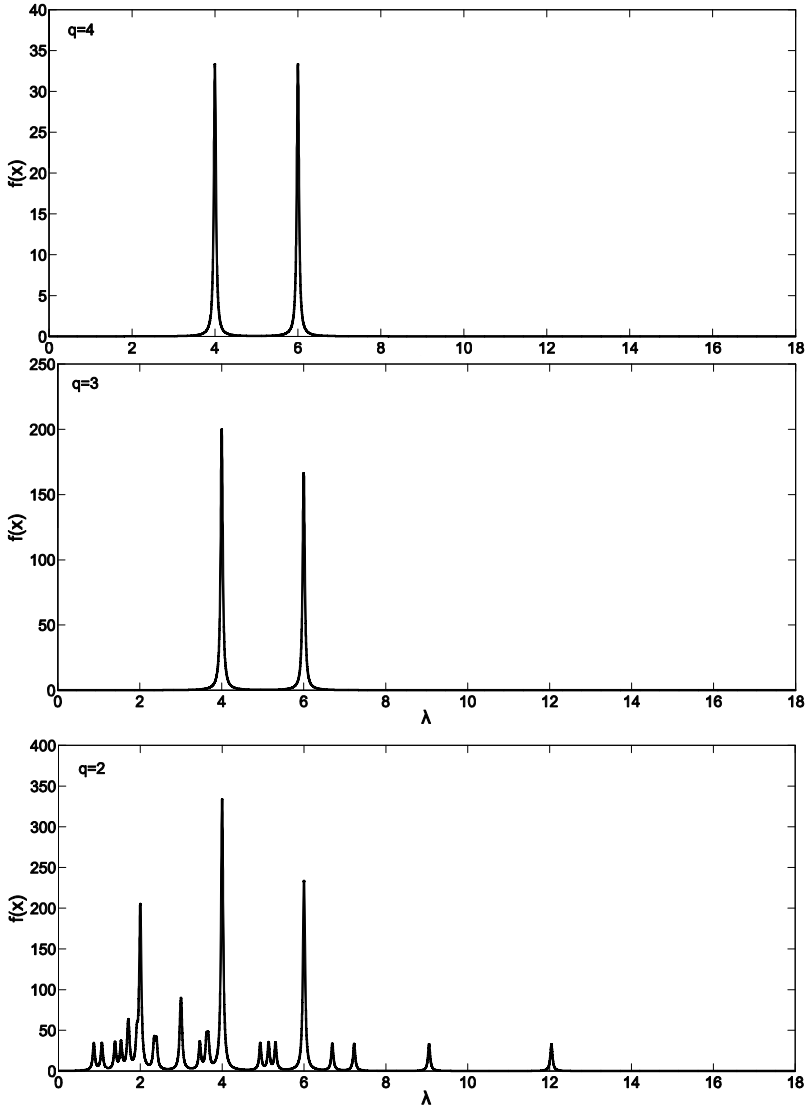
$$L_0 : C_0(K) \to C_0(K) \quad L_0 = \partial_1 \circ \partial_1^* \quad \partial_1 : C_1(K) \to C_0(K)$$

MAPS EDGES TO VERTICES

0-DIMENSIONAL COMBINATORIAL LAPLACIAN ↔ GRAPH LAPLACIAN

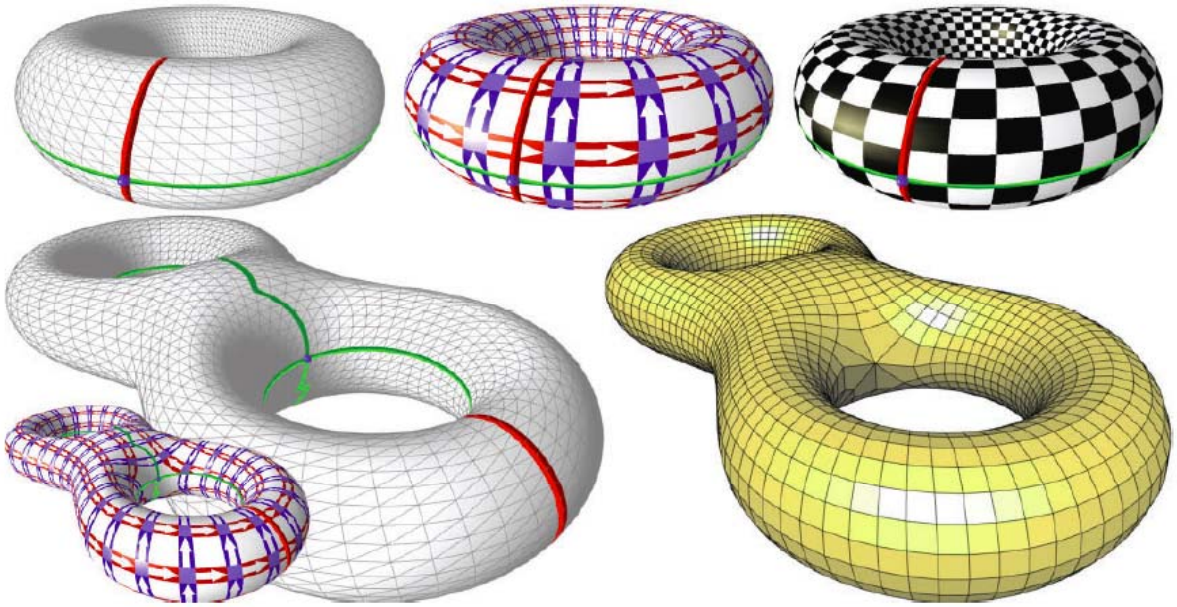
Combinatorial Laplacian of social network



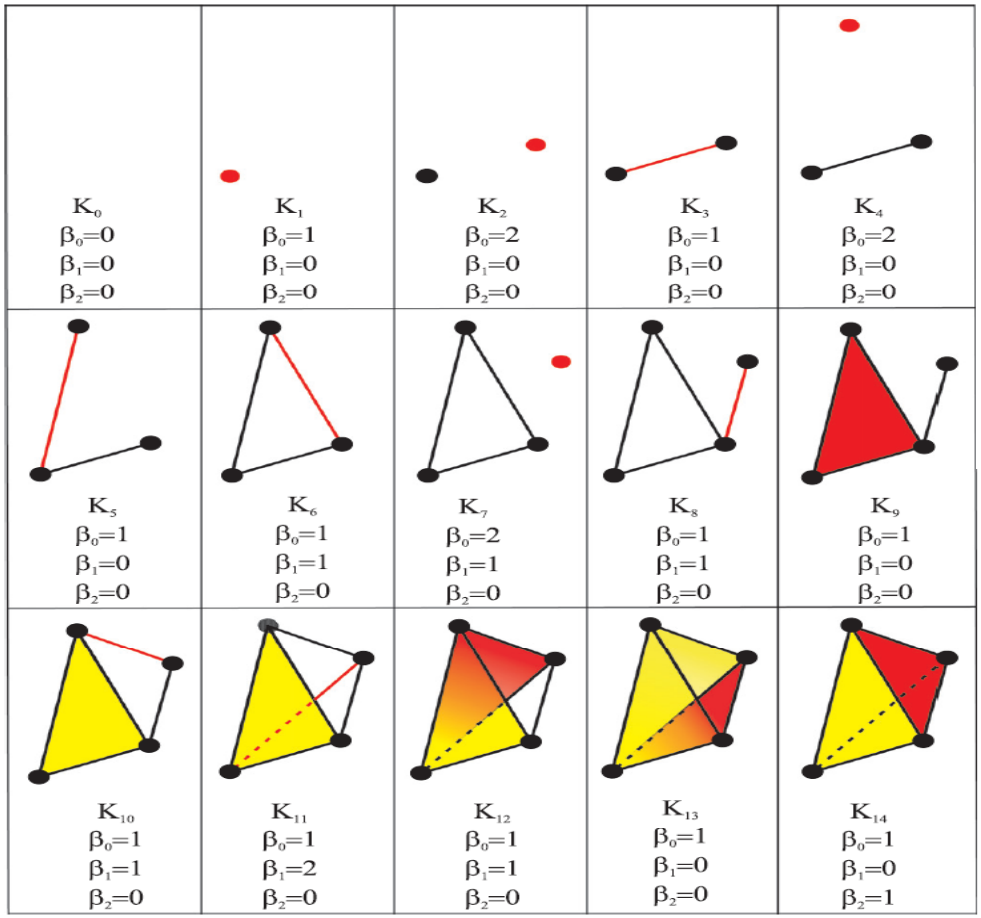


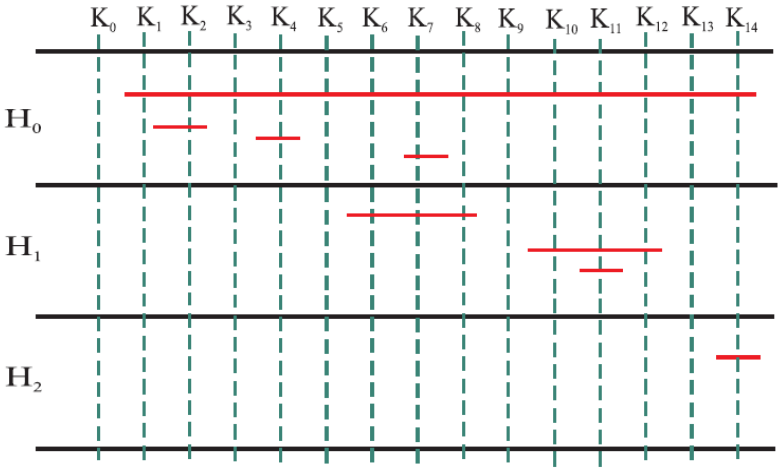
TOPOLOGICAL INVARIANT - BETTI NUMBER

$$\beta_q = \dim(H_q)$$



BETTI NUMBERS B_Q = NUMBER OF Q-DIMENSIONAL HOLES





Random networks – persistency includes H_0 and H_1

Networks with exp degree distributions – H_0, H_1 and H_2

Scale-free networks – H_0, H_1,H_2, H_3 and even H_4.

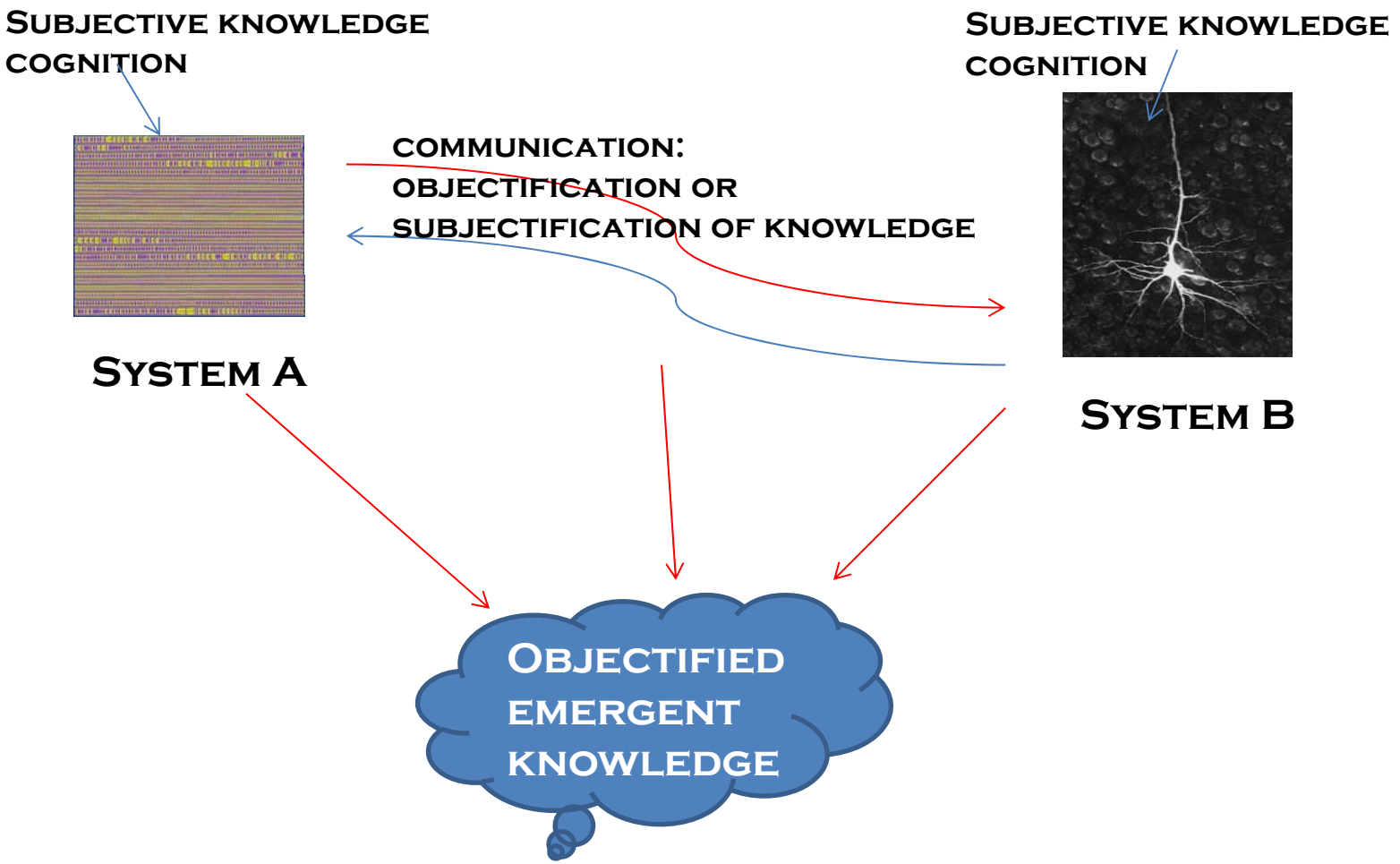
Knowledge space and simplicial complex

Knowledge moves through social ties

Innovation - new combination of knowledge

How social structures affect innovative outcomes ?

Collaboration – a way knowledge interacts and moves through social interactions which is built upon communication i.e. information flow.


SUBJECTIVE KNOWLEDGE COGNITION
SUBJECTIVE KNOWLEDGE COGNITION
COMMUNICATION: OBJECTIFICATION OR SUBJECTIFICATION OF KNOWLEDGE
SYSTEM A
SYSTEM B
OBJECTIFIED EMERGENT KNOWLEDGE

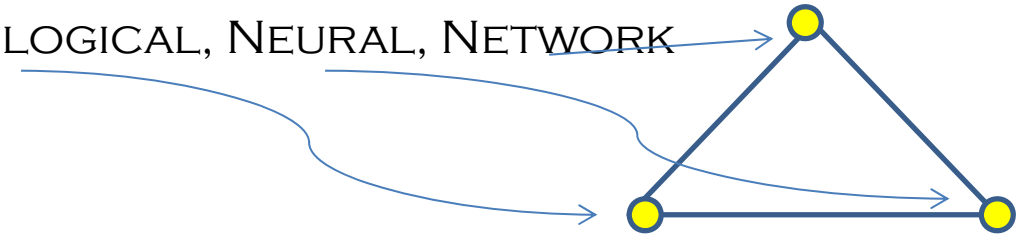
Each document consist of terms; high frequency term is a keyword representin g concept

Form a simplicial complex of concepts – an index of knowledge-based search engine

Vizulize text processing in geometric way

Vertices are keywords and edges are distances between vertices

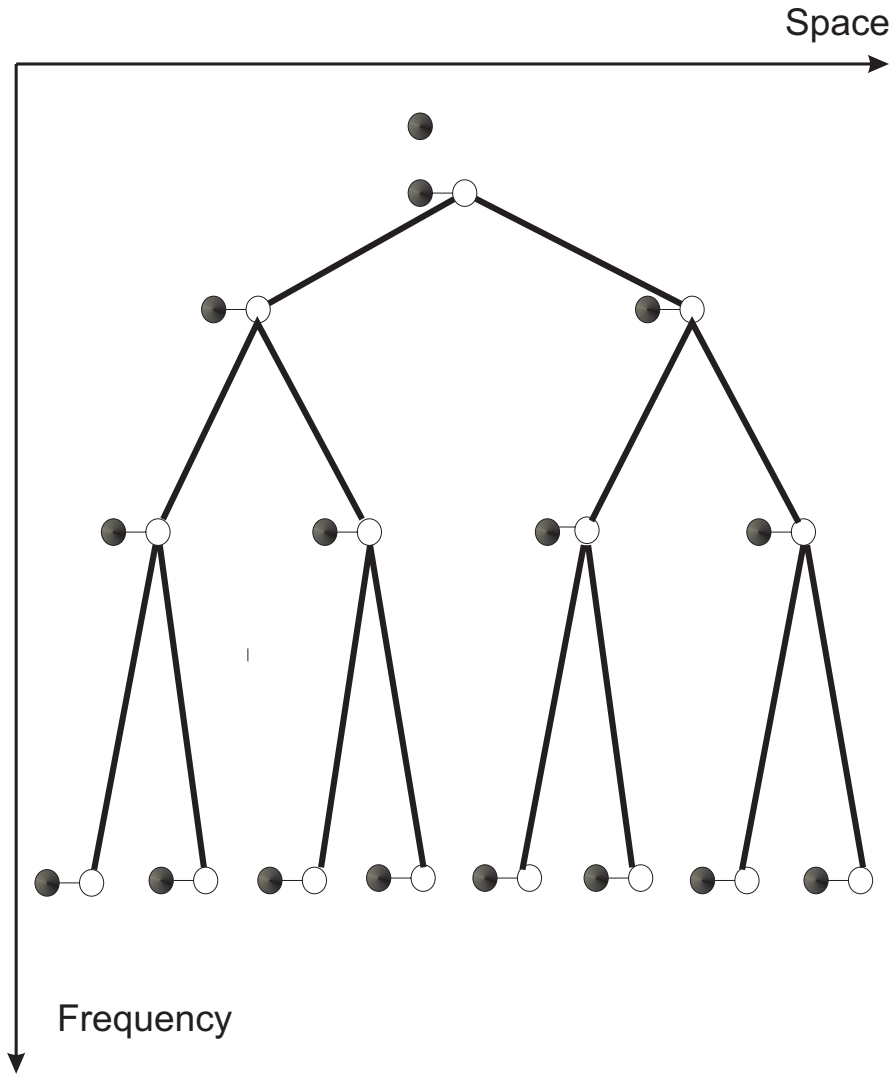
e.g. 2-simplex such as "Biological, Neural, Network

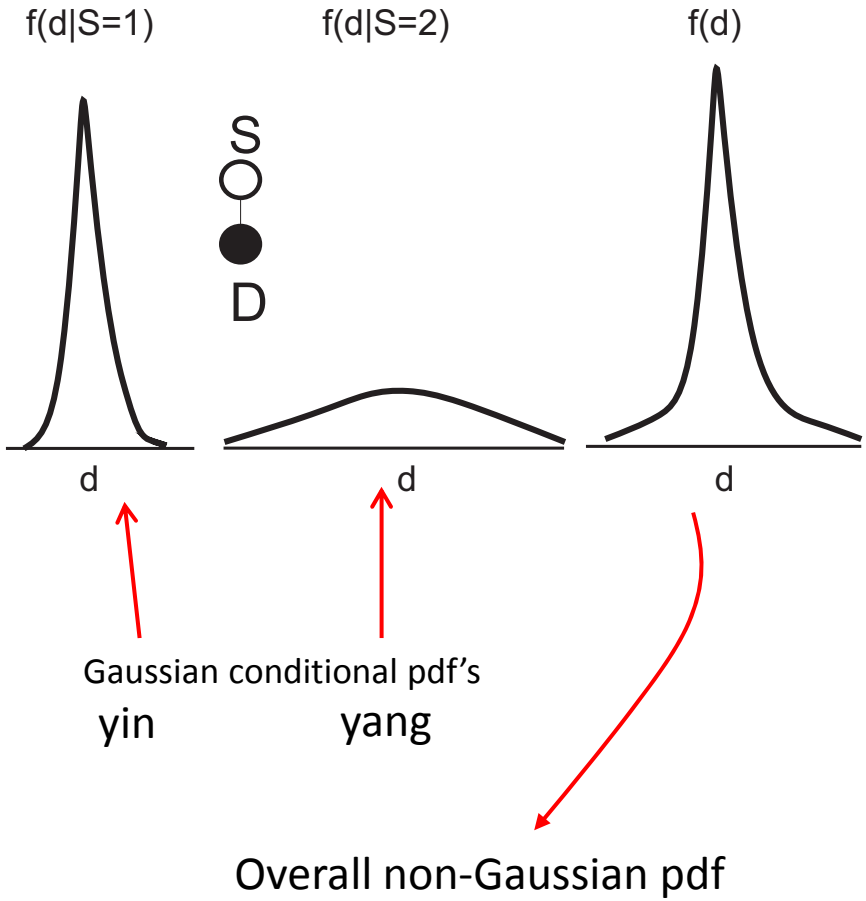


Knowledge complex – vertex order important

Different ordering of terms may present different concepts or no meaning

COMPLEXITY AND SELF-ORGANIZATION IN WAVELET NETWORK





Milovanovic & Rajkovic, Europhy. Lett. 102 (2013) 40004